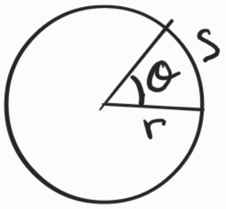
Physics 2.7 - Uniform Circular Motion **

Describing Circular Motion

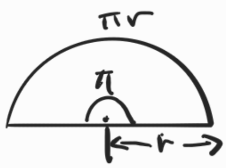
Radians (rad)  弧度



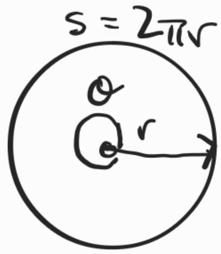
definition

$$\theta \text{ rad} = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

i.e. $1 \text{ rad} \Rightarrow s = r$

Therefore 

$\pi \text{ rad} \Rightarrow s = \pi r$



$2\pi \text{ rad} \Rightarrow s = 2\pi r$

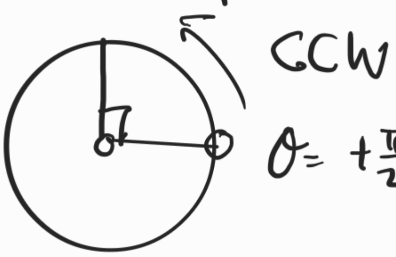
i.e. if radius $r = 1$ then $\theta \text{ rad} = s$ (arc)

Relationship between degree and radian

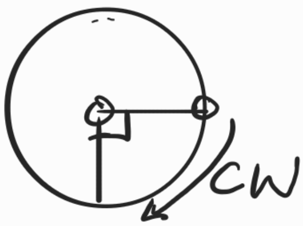
$180° = \pi \text{ rad}$ therefore $1° = \frac{\pi}{180} \text{ rad}$ and $1 \text{ rad} = \frac{180°}{\pi}$

角移位

Angular displacement (note the +ve and -ve sign)



$\theta = +\frac{\pi}{2} \text{ rad}$



$\theta = -\frac{\pi}{2} \text{ rad}$

counter clockwise by $\frac{\pi}{2}$ rad

clockwise by $\frac{\pi}{2}$ rad

Angular Velocity 角速度

$$\boxed{\omega = \frac{\theta}{t}}$$

θ ← angular displacement

t ← time taken

ω ← angular velocity

Period (T) : time taken for 1 revolution

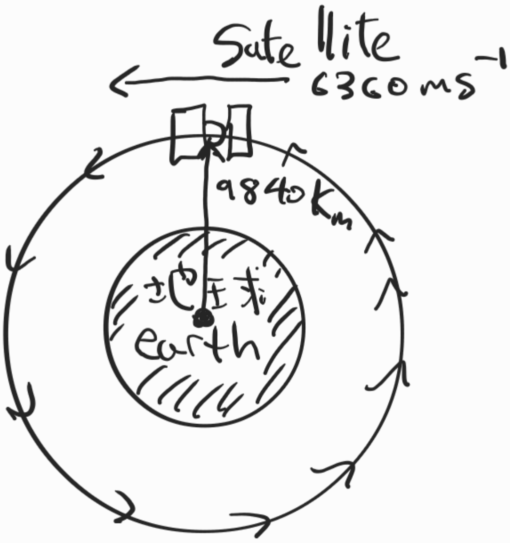
$$\boxed{\omega = \frac{2\pi}{T}}$$

Since $s = r\theta$

$$\frac{s}{t} = r\frac{\theta}{t}$$

$$\boxed{v = r\omega = \frac{2\pi r}{T}}$$

Example



a) find the angular speed and period T.

b) find the distance travelled when its orbit moves 45°.

a) $v = r\omega$

$6360 = 9840 \times 10^3 \omega$

$\omega = 6.46 \times 10^{-4}$ rad s^{-1}

$T = \frac{2\pi}{\omega}$

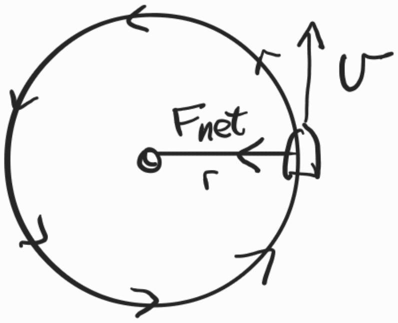
$= 9720$ s //

b) $45° = \frac{2\pi}{8}$ rad

$s = r\theta$

$= 9840 \times 10^3 \times \frac{2\pi}{8} \approx 7730$ km //

CENTRIPETAL FORCE 向心力



For uniform circular motion.
Centripetal force (F_c) occurs

$$\boxed{F_c = \frac{mv^2}{r} \quad \text{or} \quad F_c = mr\omega^2}$$

m: mass , v: linear speed , ω: angular speed , r: radius

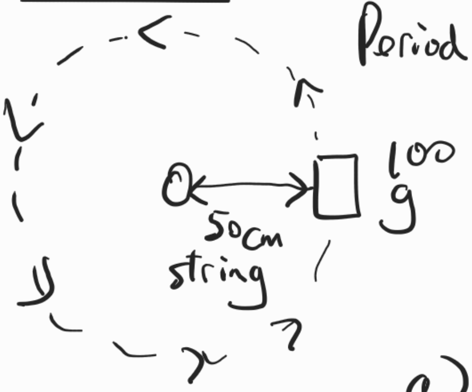
$$\boxed{a_c = \frac{v^2}{r} \quad \text{or} \quad a_c = r\omega^2}$$

Also $F_c = m a_c$

↑

centripetal acceleration 向心加速

Example



Period = 2 s

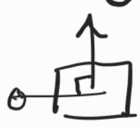
a) Find the string tension

b) If string suddenly break, what is the motion of the mass?

a) Angular speed $= \frac{2\pi}{T} = \frac{2\pi}{2} = 3.142 \text{ rads}^{-1}$

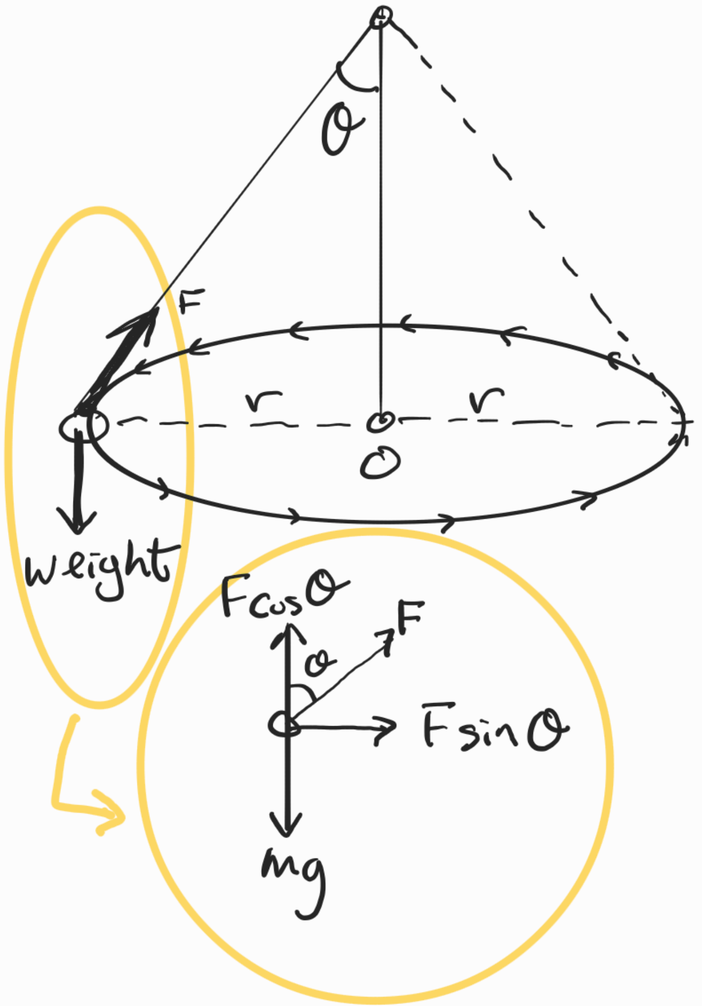
$F = mr\omega^2 = (0.1)(0.5)(3.142)$

$\simeq 0.494 \text{ N}$ //

b) If string suddenly breaks,  mass move forward linearly $\perp$ to the string.

Conical Pendulum

玩具飛機



Horizontal

$$F\sin\theta = \frac{mv^2}{r} = mr\omega^2 \quad \cdots (1)$$

Vertical

$$F\cos\theta = mg \quad \cdots (2)$$

$$\frac{(1)}{(2)} = \tan\theta = \frac{v^2}{gr} = \frac{r\omega^2}{g}$$

Example

a) Find the angular speed of plane

b) Find the string tension

Tactics

Step 1 : Identify the object + all forces

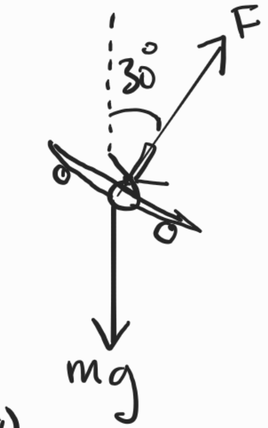
Step 2 : Resolve into X, Y directions

Step 3 : $F_{net} = mr\omega^2$

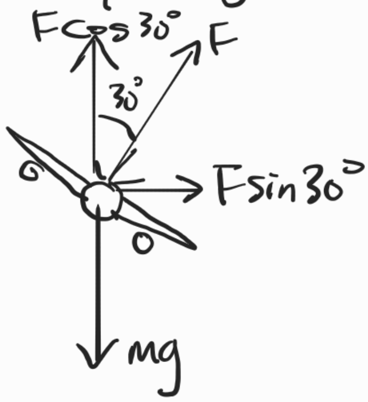
$F_{net} = 0$

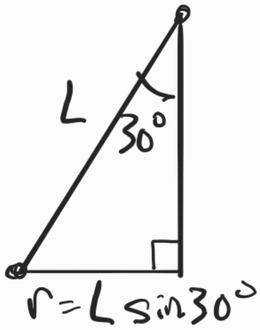
Step 4 : solve the unknowns

Solution



Free body diagram





a)

X-force, centripetal force

$$\therefore F \cancel{\sin 30^\circ} = m r \omega^2 = m(L \cancel{\sin 30^\circ})\omega^2$$

$$F = m L \omega^2 \quad \text{------} (1)$$

Y-force, the plane has no vertical motion

$$\therefore F \cos 30^\circ = mg \quad \text{----} (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\cancel{F}}{\cancel{F}\cos 30^\circ} = \frac{\cancel{m}L\omega^2}{\cancel{m}g}$$

$$\frac{1}{\cos 30^\circ} = \frac{(1)\omega^2}{9.81} \Rightarrow \omega \approx 3.37 \text{ rads}^{-1}$$

b) Tension in string

$$F = mL\omega^2 = (0.1)(1)(3.37)^2 \approx 1.13 \text{ N}$$

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