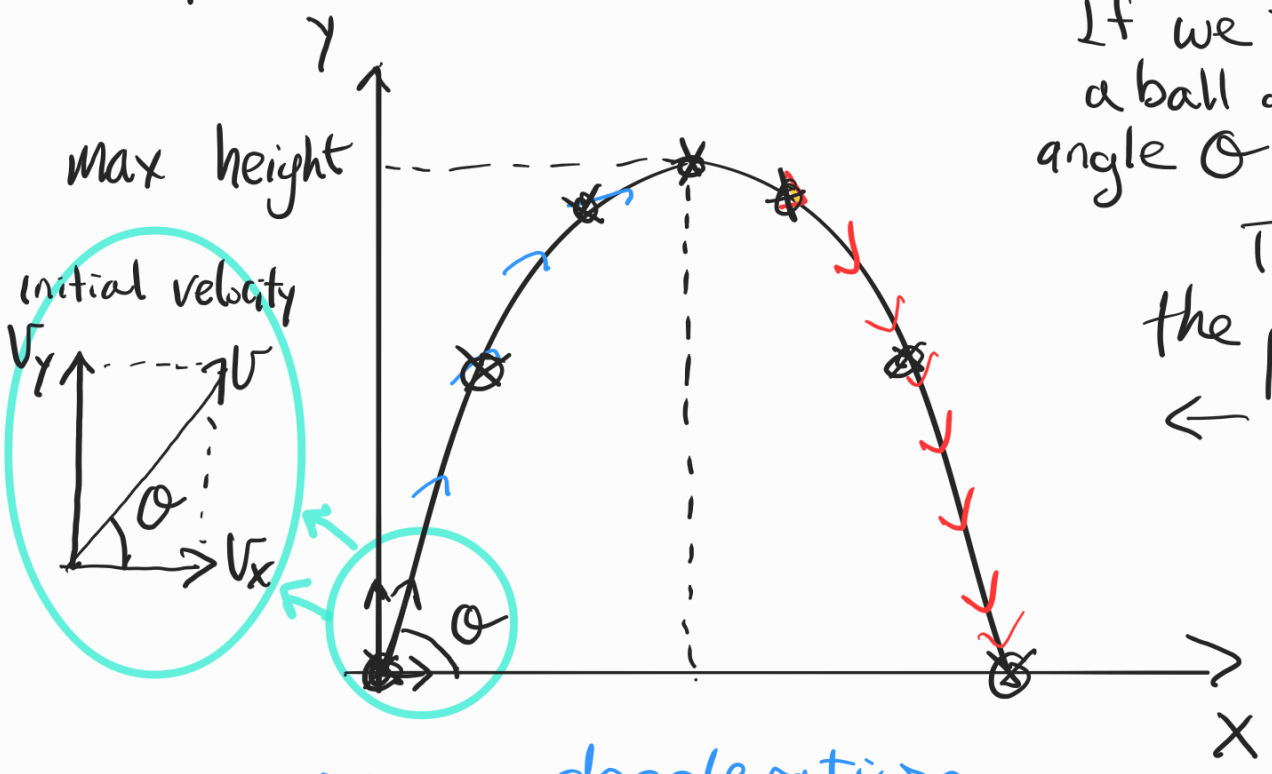
2.6 - Projectile Motion ** 拋體運動

Independent x- y motions

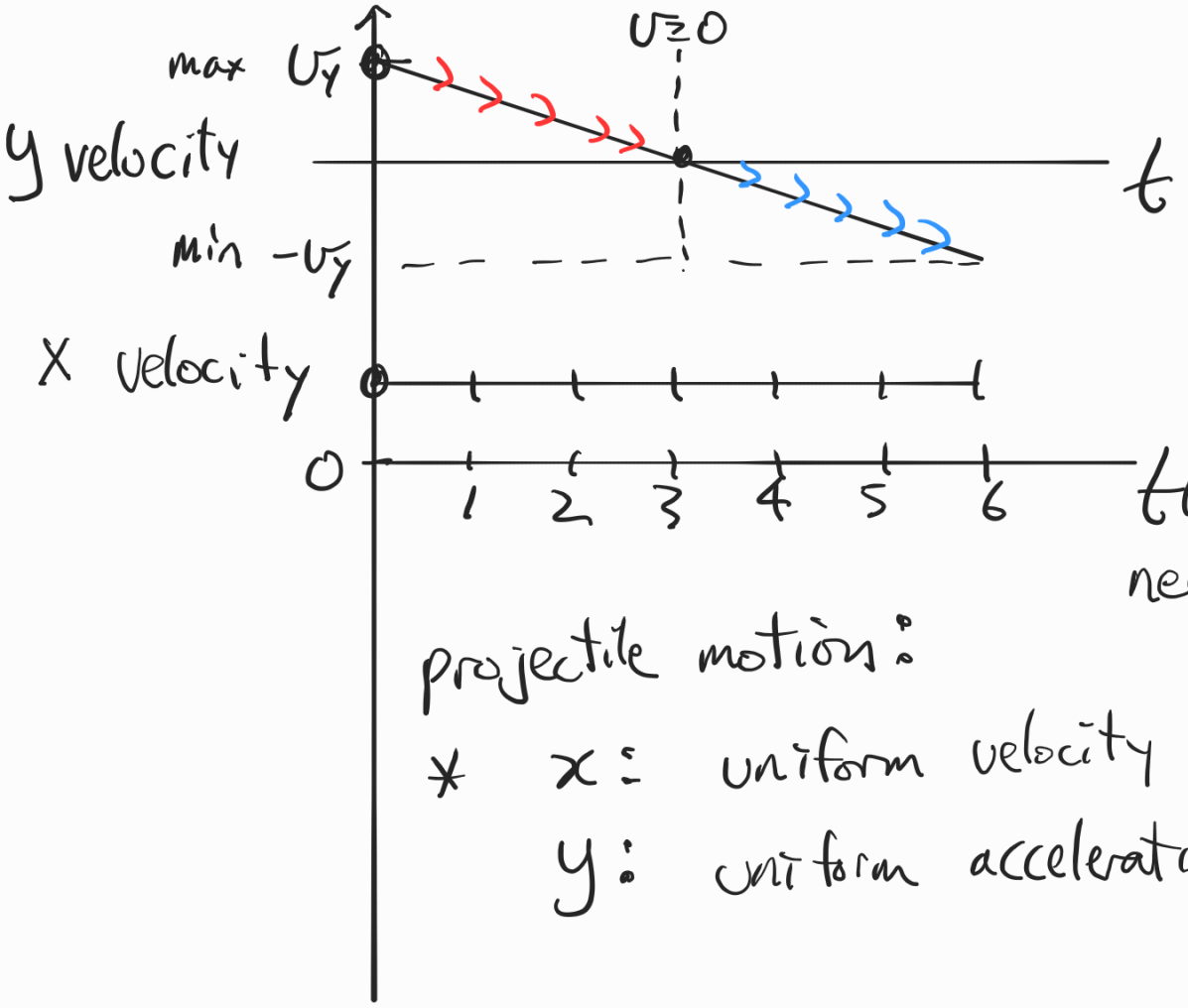


If we throw a ball at an angle θ

This is the projectile ← 軌跡

→→→ deceleration

→→→ acceleration



$a_y = -9.81\ ms^{-2}$

$a_x = 0$

v_x = constant

neglect air resistance

projectile motion:

* x: uniform velocity

y: uniform acceleration

For x motion: $(u_x = u\cos\theta)$

$x = u_x t$

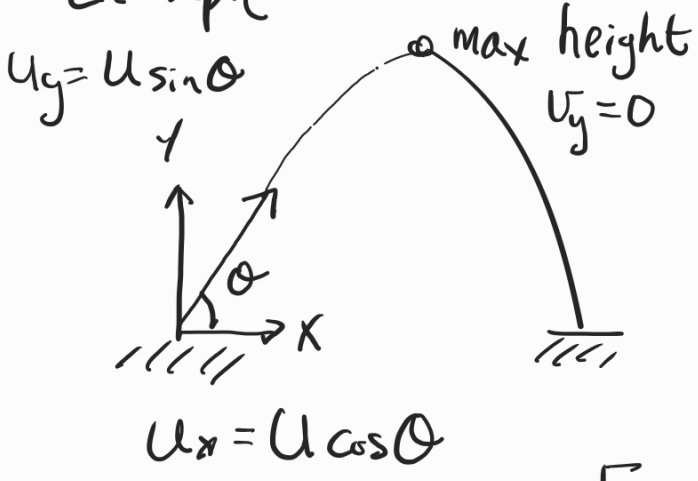
$v_x = u_x$

$a_x = 0$

For y motion: $(u_y = u\sin\theta)$

$$\left.\begin{aligned} y &= \tfrac{1}{2}(u_y + v_y)t \\ y &= u_y t + \tfrac{1}{2} a_y t^2 \\ v_y &= u_y + a_y t \\ v_y^2 - u_y^2 &= 2 a_y y \end{aligned}\right\} a = -g$$

Example



Find max height:

$v_y^2 - u_y^2 = 2 a_y y$

$0 - u_y^2 = 2(-g)H$

Find time of flight

$y = u_y t + \tfrac{1}{2} a_y t^2$

same initial level ⟶ $0 = u_y t + \tfrac{1}{2}(-g)t^2$

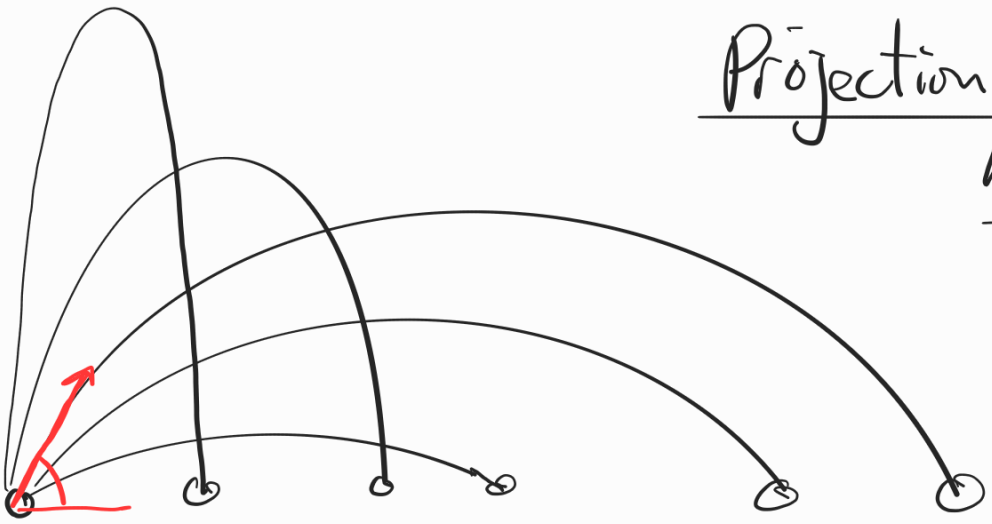
$$T = \frac{2u_y}{g} \quad //$$

Find distance travelled: $x = T\, u_x$

$= T\, u\cos\theta$

Projection Range and Angle

(neglect air resistance)



θ 45° when $\theta = 45^\circ$ max range is achieved.

$$R_{max} = \frac{u^2}{g}$$

$$R_{max} = \frac{u^2 \sin(2\theta)}{g}$$

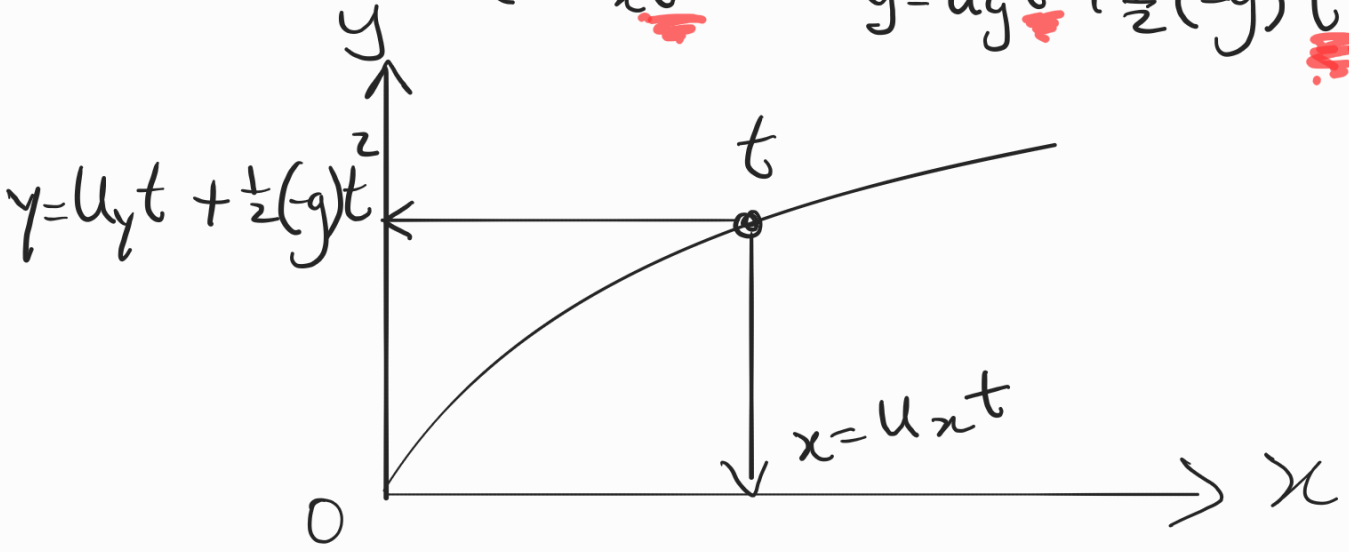
} max value of $\sin\theta = 1$ at 90°
$\therefore$ for 2θ, $\theta = 45^\circ$

To sum up, we use t as the middle man

$x \rightleftarrows t \rightleftarrows y$

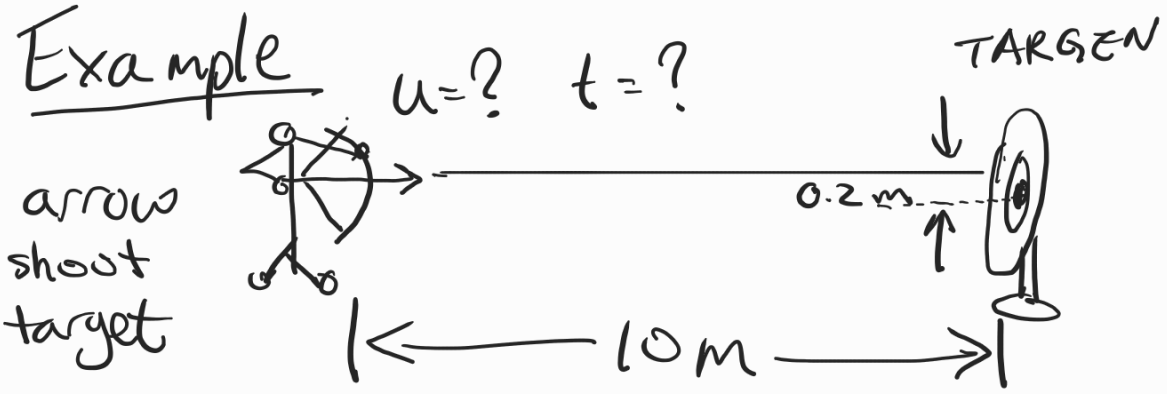
$x = u_x t$

$y = u_y t + \frac{1}{2}(-g)t^2$



Example

$u = ?$ $t = ?$



a) Find t:

$$y = u_y t + \frac{1}{2} a_y t^2 \quad \text{(use } y \text{ axis)}$$

$$0.2 = 0 + \frac{1}{2}(9.81)t^2$$

$t = +0.202\,s$ // or ~~$-0.202\,s$~~ rejected

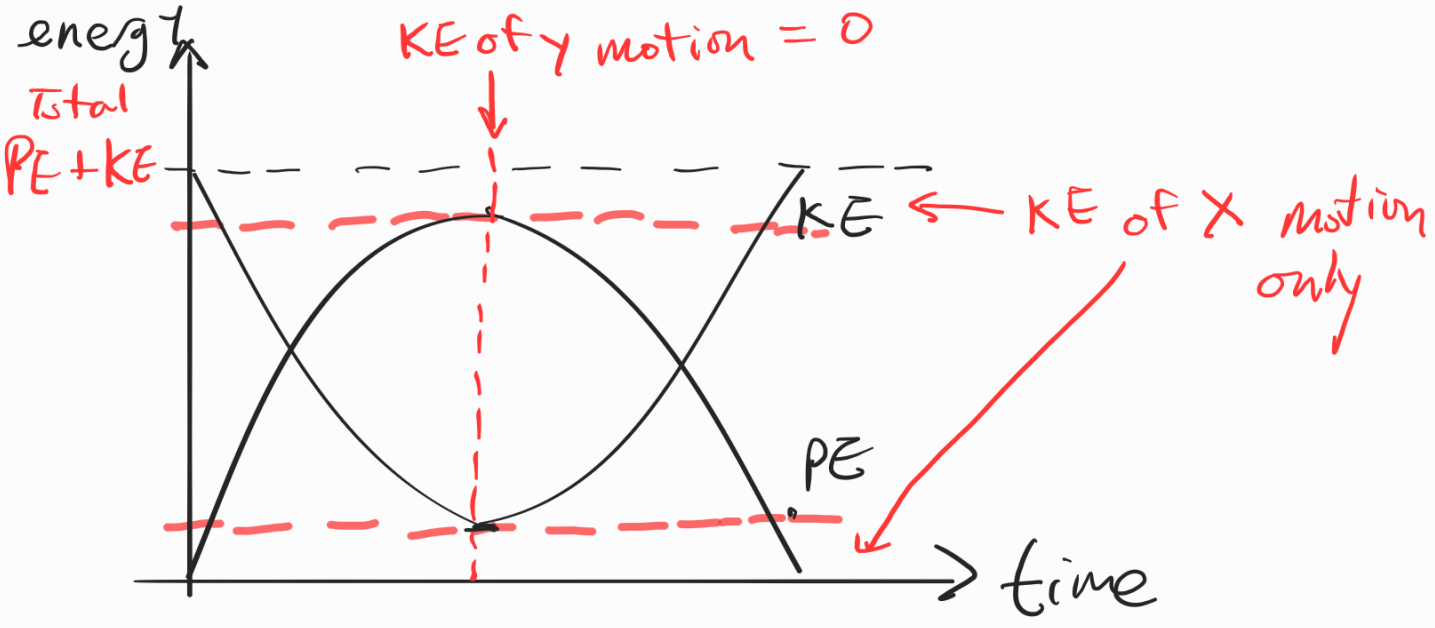
b) Find u:

$$x = u_x t$$

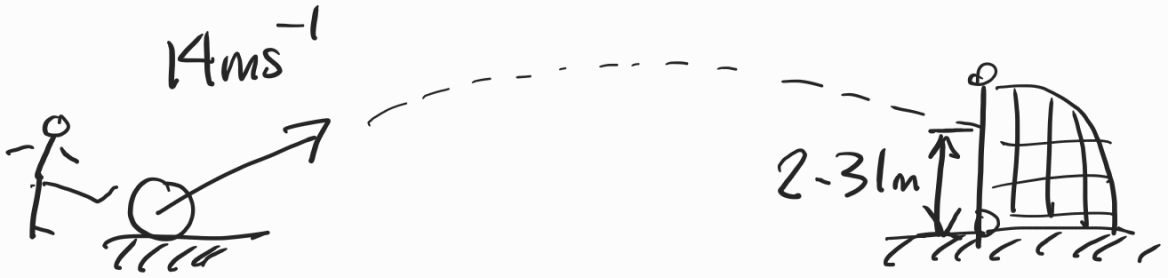
$$10 = u(0.202)$$

$$u = 49.5\,ms^{-1}$$ //

Projectile and PE + KE



Example: Find the speed of ball enters the goal



initial KE + initial PE = final KE + final PE

$$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + mgy$$

$$\frac{1}{2}14^2 = \frac{1}{2}v^2 + 9.81 \times 2.31$$

$$v = 12.27 \approx \underline{12.3\ ms^{-1}}$$

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