20 Arithmetic and Geometric Sequences

Review

Sequence : List of numbers e.g. 1, 2, 3, 4, --- (term: 1; sequence: 1, 2, 3, 4, ---)
Term : No in the sequence ½, ⅓, ¼, ----
General Term : $T(n) = n + 7$
n^{th} term $= 8, 9, 10, 11$ ----
($n=1$: 8; $n=2$: 9; $n=3$: 10)

等差數列

Arithmetic Sequences Difference between any 2 successive terms are the same

e.g. 3, 11, 19, 27, 35
diff: 8, 8, 8, (8) → common difference 公差

equation : $\boxed{T(n) = a + (n-1)d}$
(a: 1st term; n: n^{th} term; d: common difference)

e.g. $a = 3$, $d = 2$ ⇒ sequence 3, 5, 7, 9 ----

equation : $\boxed{T(n) = \frac{1}{2}[T(n-1) + T(n+1)]}$
($T(n)$: value of n^{th} term; $T(n-1)$: value before n^{th} term; $T(n+1)$: value after n^{th} term)

3, 5, 7, 9, 11
$T(n-1)$ = 5, $T(n+1)$ = 9 ⇒ $(5+9) \div 2 = 7$

Property of Arithmetic Sequence :

If $T(1), T(2), T(3),$ ------ is an arithmetic seq.
Then $kT(1)+h, kT(2)+h, kT(3)+h$ ---- is also A seq.

example: $T(1), T(2), T(3),$ --- ⇒ 1, 3, 5, 7, 9,
$k=2$ $h=3$ $kT(1)+h, kT(2)+h, kT(3)+h$ --- ⇒ 5, 9, 13, 17

Geometric Sequences
等比數列

$$T(n) = a r^{n-1}$$

n^{th} term　first term　common ratio 公比

$T(1) = a$
$T(2) = T(1) \times r = ar$
$T(3) = T(2) \times r = ar^2$
$T(4) = T(3) \times r = ar^3$

eg.: $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}$

$a = 1 \quad r = -\frac{1}{3}$

equation: $T(n)^2 = T(n-1) \times T(n+1)$

example: $T(n)^2 = 15 \times 60 = 900$

$T(n) = \underline{-30 \text{ or } 30}$

sequence: ---- 15, 30, 60 --- or --- 15, -30, 60 ---

$r = 2$　　$r = -2$

Property of Geometric Sequence

If $T(1), T(2), T(3), \cdots$ is geometric sequence

then $kT(1), kT(2), kT(3) \cdots$ is also a geometric sequence　$k \neq 0$

Example: $T(1), T(2), T(3), \cdots \Rightarrow 1, 2, 4, 8 \quad r = 2$

$kT(1), kT(2), kT(3) \cdots \Rightarrow 2, 4, 8, 16 \quad k = 2 \quad r = 2$

Applications of Arithmetic and Geometric Sequences

Example 1: In a lecture theatre, number of seats in n^{th} row form an arithmetic sequence. When $n = 5$, $T(n) = 20$. Each row has 2 more seats than the row in front.

(a) Find the number of seats in 14^{th} row

answer: $T(5) = 20 \quad d = 2$

$T(n) = a + d(n-1) \Rightarrow a + 2(5-1) = 20, \quad a = 12$

$T(14) = 12 + 2(14-1) = 38$ //

(b) There are 60 seats in the m^{th} row, what is m?

$T(m) = 60 \Rightarrow 12 + 2(m-1) = 60$

$m = 25$

Example 2: The value of a new equipment is \$6000. After one year, its value falls to \$5700, and after two years, its value falls to \$5415

(a) Show that the value fall is a geometric sequence

answer: 6000, 5700, 5415

$\frac{T(2)}{T(1)} = \frac{5700}{6000} = \frac{19}{20}$, and $\frac{T(3)}{T(2)} = \frac{5415}{5700} = \frac{19}{20}$

Therefore $a = 6000$, $r = 19/20$, geometric sequence.

(b) Find the value after it has been used for 6 years

answer: after 6 years, it is on the 7^{th} term $n = 7$

$T(7) = 6000 \times \left(\frac{19}{20}\right)^{7-1} = \underline{\$4410.60}$

(a = 6000, r = 19/20)