19. More about Probability

Review

Definition of Probability (e.g. throw a dice)

$$P(E) = \frac{\text{number of outcomes favourable to } E}{\text{number of outcomes in sample space } S} = \frac{3}{6} = \frac{1}{2}$$

P : Probability
E : Event number
S : Sample Space number

$P(E) = 0$ impossible
$P(E) = 1$ certain
$0 \leq P(E) \leq 1$ range of P

Experiment Probability (e.g. experiment of getting an even no.)

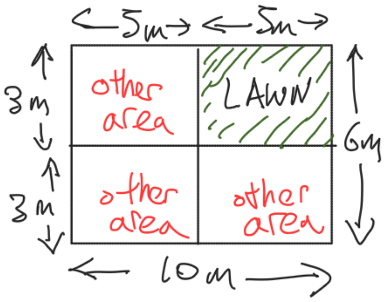
$$P(A) = \frac{\text{number of times event } A \text{ occurs}}{\text{total number of trials}}$$

If outcome : 2 (x15 times), 4 (x10 times), 6 (x9 times)

Total tries : x60 times

$$P(A) = \frac{15 + 10 + 9}{60} = \frac{17}{30}$$

Geometry Probability (e.g. probability ball will go to lawn)



$$P(E) = \frac{\text{area of region in which } E \text{ occurs}}{\text{area of whole garden}}$$

$$= \frac{5 \times 3}{10 \times 6} = \frac{1}{4}$$

SET Language in Probability

$S = \{1, 2, 3, 4, 5, 6\}$

a set S — contains the following elements

$A = \{2, 4, 6\}$

outcome of event — elements of outcome

$n(X) = 6$

number of elements in a set 'X'

$n(\emptyset) = 0$ ← no element

$\emptyset$ set or 'empty set'

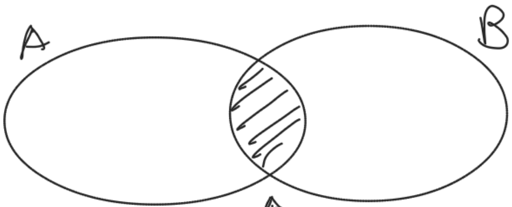
Example $S = \{S, E, N, I, O, R\}$ full set

$C = \{E, I, O\}$ contains vowels

$n(S) = 6 \quad n(C) = 3$

Venn Diagrams

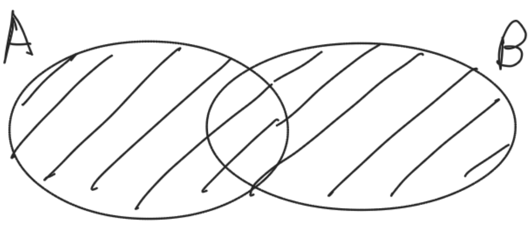
A B



intersection

intersection of A, B $(A \cap B)$
contains elements belong to both A and B

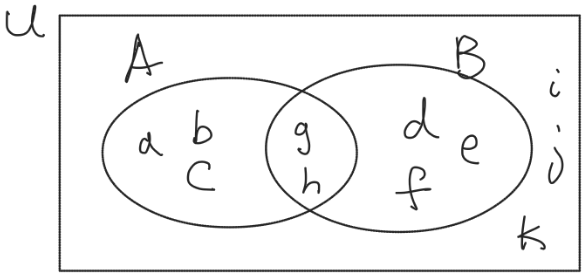
A B



union

Union of A, B $(A \cup B)$
contains elements belong to both A or B

Example

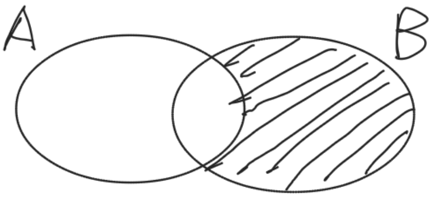


$A \cap B = \{h, g\}$

$A \cup B = \{a, b, c, d, e, f, g, h\}$

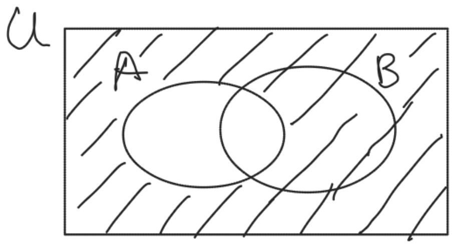
enclosed by — but not include

Complement of A in B ($B \setminus A$)

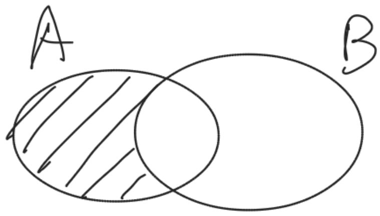


Contains elements in B which are **NOT** in A

$B \setminus A$ ← 不在A

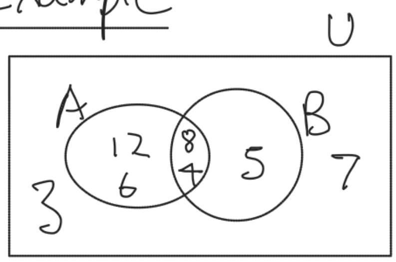


A' $U \setminus A$ ← 不在A



$A \setminus B$ ← 不在B

Example



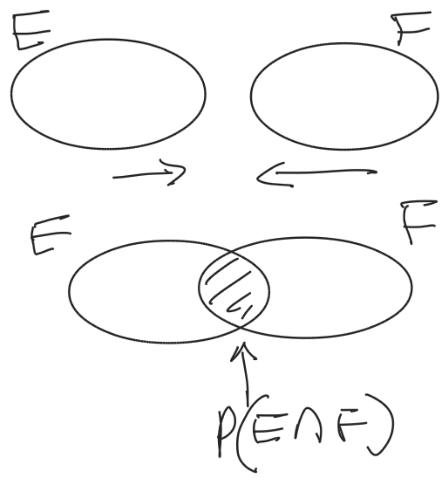
$A \setminus B = 1, 2, 6$
$B \setminus A = 5$
$U \setminus A = 3, 5, 7$
$U \setminus B = 3, 1, 2, 6, 7$

Use Set Language in Probability

$$P(E) = \frac{\text{no. of outcomes favourable to } E}{\text{total no. of possible outcomes}}$$

$$= \frac{\text{no. of elements in event } E}{\text{total no. of elements in space } S} = \frac{n(E)}{n(S)}$$

Addition law of Probability



For 2 events E and F

$$P(E \overset{\text{or}}{\cup} F) = P(E) + P(F) - P(E \underset{\text{and}}{\cap} F)$$

Mutually Exclusive Event - event that cannot occur at the same time

e.g. When throwing a dice

→ * getting an odd number and an even number

X * getting an even number and number > 3 (possible)

For mutually exclusive event: $P(E \cup F) = P(E) + P(F)$

$E \cap F = \varnothing$

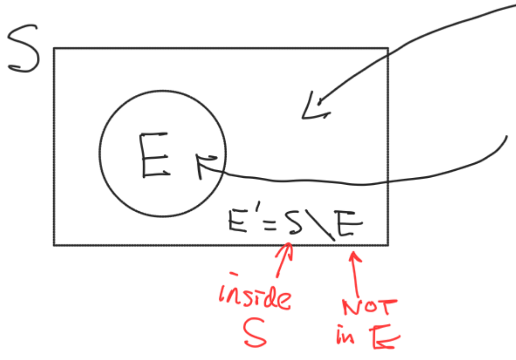
$P(E \cap F) = 0$

$n(E \cap F) = 0$

This can be extended as

$$P(E_1 \overset{\text{union}}{\cup} E_2 \cup \text{-----} E_n) = P(E_1) + P(E_2) \text{--} P(E_n)$$

Event plus Complemetary Event = 1



$$P(E) + P(\overset{\text{complematary}}{E'}) = 1$$

or

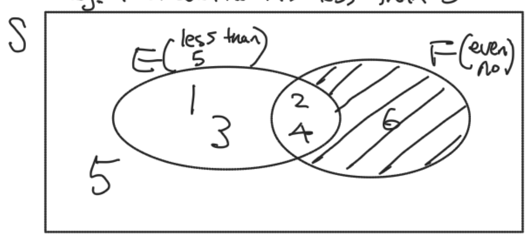
$$P(E') = 1 - P(E)$$

Multiplication Law of Probability

Conditional Probability 條件概率

Two events F, E :

e.g. find even numbers less than 5

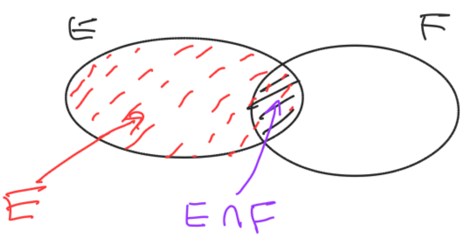


conditional probability of F, given that E has occured

answer must be inside E

☆ E has occured

☆ Must fulfil E and F



$$\underbrace{P(F \mid E)}_{\text{condition probability of F, given that E has occured}} = \frac{n(E \cap F)}{n(E)}$$

fits both E and F

result out of E

Dependent Events (e.g. draw two colour balls from the same bag)

Probability of second event, is affected by the first event

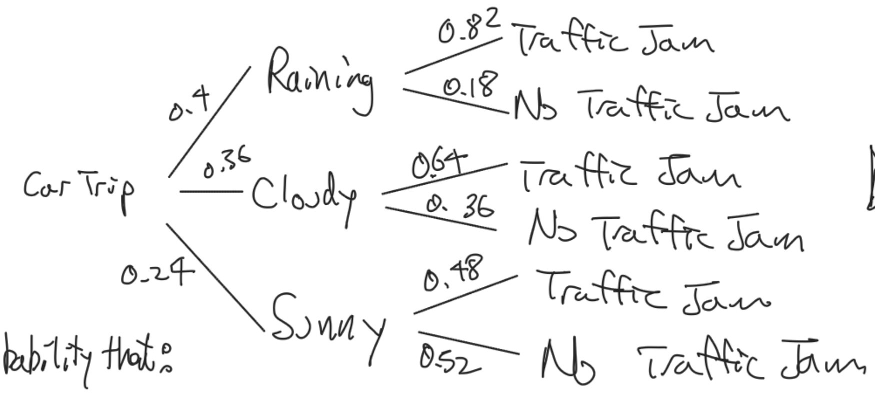
Independent Events (e.g. draw two colour balls from 2 separate bags)

Probability of second event is not affected by the first event.

$P(E|F) = P(E)$ E, F are two separate events

Equation: $$P(E_1 \underset{\text{and}}{\overset{\text{intersection}}{\cap}} E_2 \cap E_3 \cdots \cap E_n) = \underbrace{P(E_1)}_{\text{individual probability}} \times P(E_2) \times P(E_3) \times \cdots P(E_n)$$

Example



TREE DIAGRAM

Find the Probability that:

a) it will rain and there is traffic jam

b) There is traffic jam

c) it will rain, given that here is traffic jam

a) $P(\text{rain}) \times P(\text{traffic jam}) = 0.4 \times 0.82 = \underline{\underline{0.328}}$

b) $P(\text{traffic jam}) = P(\text{rain and traffic jam}) + P(\text{Cloudy and traffic jam} + P(\text{Sunny and traffic jam})$

$= (0.4 \times 0.82) + (0.35 \times 0.64) + (0.25 \times 0.48)$

$= \underline{\underline{0.672}}$

c) $P(\text{it will rain, given that there is traffic jam})$

$= \dfrac{P(\text{it will rain and there is traffic jam})}{P(\text{there is traffic jam})}$

$= \dfrac{0.328}{0.672} = \dfrac{41}{84}$

Solving Probability Problems by Permutation and Combination

Example 1 : 5 children are queuing for drinks. Find the probability that two of the children, are first and second in the queue (consider order)

Solution : Probability the other 3 children in 3rd, 4th, 5th position : P^3_3

Total number of arrangement for 5 children : P^5_5

Probability is $= \frac{P^3_3}{P^5_5} = \frac{1}{20}$

Example 2 : 10 boys and 6 girls are chosen at random to form a committee. Find the probability that 3 boys and 1 girl in a committee (no order)

Solution : No of ways to select 3 boys and 1 girl

$= C^{10}_3 \times C^6_1$

No of ways to select 4 committee members

$= C^{16}_4$

$\therefore P(3 \text{ boys and } 1 \text{ girl}) = \frac{C^{10}_3 \times C^6_1}{C^{16}_4} = \frac{36}{91}$

Example 3: A 5 digit number is formed from $\{1, 3, 5, 6, 7, 8, 9\}$. If each digit can be used once, find the probability that:

a) The number consists of digits not less than 5 (ie 5 or more)
b) is less than 50,000.

Solution:

a) There are 5 digits: 5, 6, 7, 8, 9 $\Rightarrow P_5^5$

Total no of ways to form a 5 digit number $\Rightarrow P_5^7$

$$\text{Probability} = \frac{P_5^5}{P_5^7}$$

b) For a number less than 50,000

The first digit should be 1 or 3 (i.e. 1xxxx or 3xxxx)

Number of ways to choose 1st digit $= C_1^2$ (no order)

Number of ways to choose remaining digits (xxxx) out of 6 digits $= P_4^6$

$$P = \frac{C_1^2 \times P_4^6}{P_5^7} = \underline{\underline{\frac{2}{7}}}$$

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