

18 Permutation and Combination (排列 and 組合)

Addition Rule OR

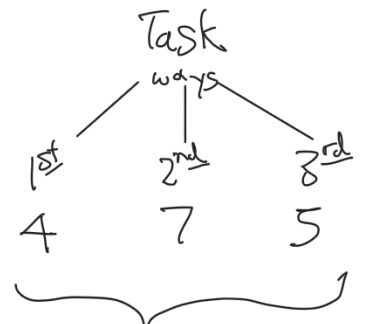
- 4 red circle
- 7 blue circle
- 5 green circle

total 16 circles

To select one circle from the group:

- 4 reds 1st possibility
- 7 blues 2nd possibility
- 5 greens 3rd possibility

← 16 ways



total number of tasks = $4 + 7 + 5 = 16$

- If
- R_1 : 1st possibility
 - R_2 : 2nd possibility
 - $\vdots$
 - R_k : kth possibility

Total ways of selection = $R_1 + R_2 + \dots + R_k$

Example: Wants to select a student who can speak French or Spanish

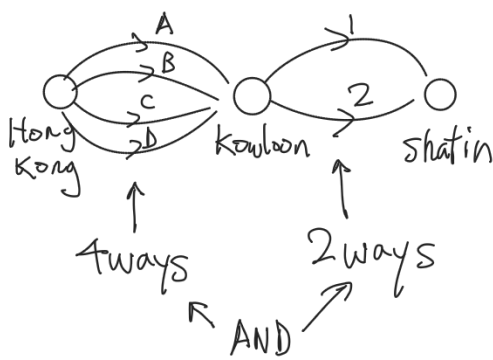
145 speaks Spanish
 231 speaks French

} out of this, there are 50 students who can speak both languages

Number of choices: $145 + 231 - 50 = 326$

(-50 is to prevent double counting)

Multiplication Rule AND



Note: 2 subtasks are totally independent

How many ways you can travel from HK → Shatin?

$4 \times 2 = 8$ ways

(A1, A2, B1, B2, C1, C2, D1, D2)

Total number of ways to complete the task: $R_1 \times R_2 \times \dots \times R_j$

where R_1 : different ways for first step; R_2 : different ways 2nd step; R_j : different ways jth step

Combination

* selection of objects without considering the order (不分先後)

$$* C_r^n = \frac{P_r^n}{r!} = \frac{n!}{r!(n-r)!}$$

Previous Example: A, C, E, G select 2 and arrange in order (分先後)

Answer (12 cases): AC, CA, AE, EA, AG, GA, CE, EC, CG, GC, EG, GE

Combination (不分先後): AC, AE, AG, CE, CG, EG (6 cases)

$$C_2^4 = \frac{P_2^4}{2!} = \frac{12}{2 \times 1} = \underline{\underline{6}}$$

Example (兩食店) A fast food restaurant offer 7 different dishes. A customer can choose 1, 2, or 3 different dishes of food. How many different ways can a customer choose?

Solution

$$\left. \begin{array}{l} 1 \text{ dish only} : C_1^7 \\ 2 \text{ dishes} : C_2^7 \\ 3 \text{ dishes} : C_3^7 \end{array} \right\} \text{total choice} = C_1^7 + C_2^7 + C_3^7 = \underline{\underline{63}}$$

