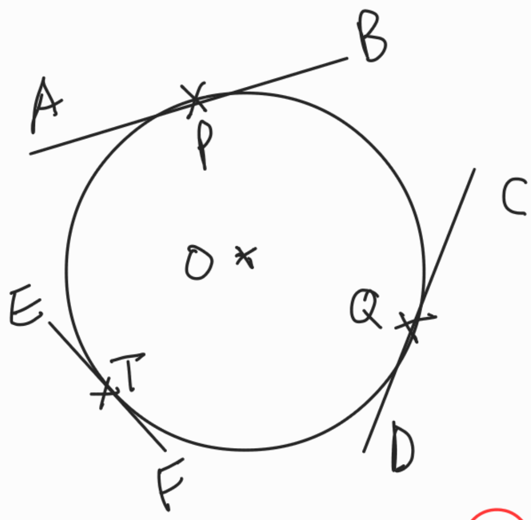
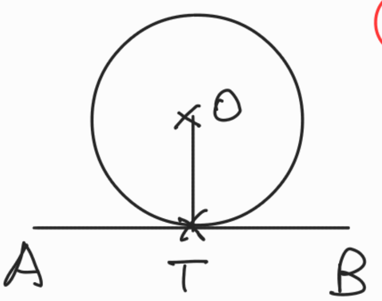
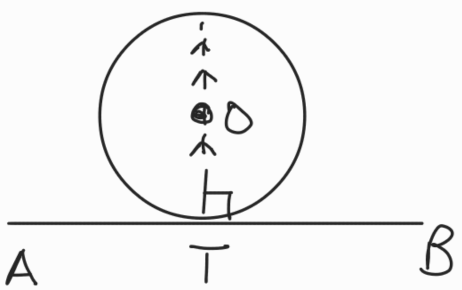
15 Tangents to Circles



AB, CD, EF are tangents to the circle. Point of contact : P, Q, T



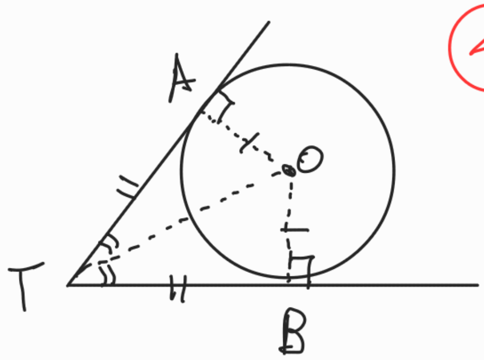
1 * If AB is tangent to circle, AB ⊥ OT
(tangent ⊥ radius)



2 * If AB ⊥ OT, then AB is tangent to the circle
(converse of tangent ⊥ radius)

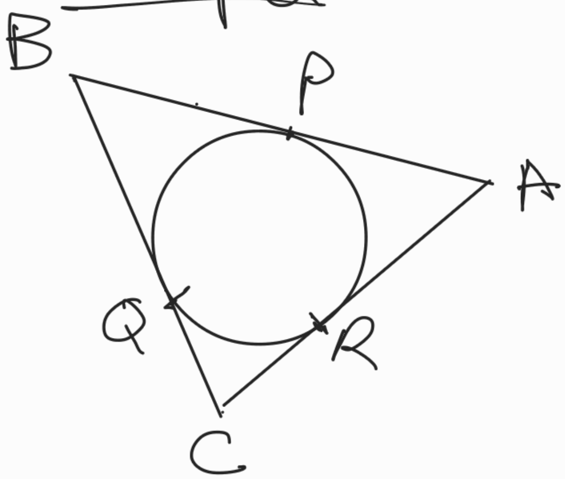
3 The ⊥ to a tangent passes through centre of circle
(point of contact of tangent, its ⊥ passes through O)

Tangents to a circle, from an external point



4 If 2 tangents are drawn to form connection point T, then
(a) $TA = TB$
(b) $\angle TOA = \angle TOB$
(c) $\angle OTA = \angle OTB$
(Tangent properties)

Example.



$$\left.\begin{aligned} AB &= 20\text{cm} \\ BC &= 12\text{cm} \\ CA &= 16\text{cm} \end{aligned}\right\} \text{find } QC$$

$$\begin{aligned} BQ &= BC - QC \\ &= 12\text{cm} - QC \end{aligned}$$

$$\begin{aligned} BP &= BQ \quad (\text{tangent properties}) \\ &= 12\text{cm} - QC \text{ ------ } (1) \end{aligned}$$

$RC = QC$ (tangent properties)

$AR = AC - QC = 16\text{cm} - QC$

$$\begin{aligned} AP &= AR \quad (\text{tangent properties}) \\ &= 16\text{cm} - QC \text{ ----- } (2) \end{aligned}$$

From (1) and (2):

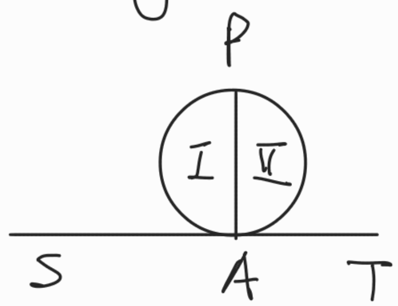
$$\begin{aligned} AB &= BP + AP \\ &= (12\text{cm} - QC) + (16\text{cm} - QC) \\ &= 28\text{cm} - 2QC \end{aligned}$$

$$20\text{cm} = 28\text{cm} - 2QC$$

$$\therefore QC = 4\text{cm} \; //$$

圓周角 交錯弓形

Angles in the Alternative Segments



Segment I is the alternative segment to $\angle PAT$

Segment II is the alternative segment to $\angle PAS$

$\angle PAT$ and $\angle PAS$ are <u>tangent-chord angles</u>
弦切角

A Tangent - Chord angle of a circle is equal to any angle in the alternative segment

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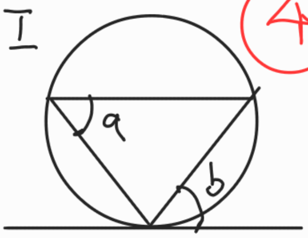
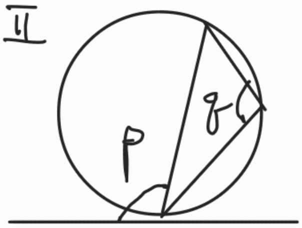
I



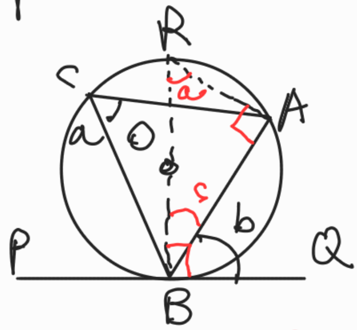
II



$a = b$ $p = q$

(∠ in alt segment)

proof I



$a + c + 90^\circ = 180^\circ$

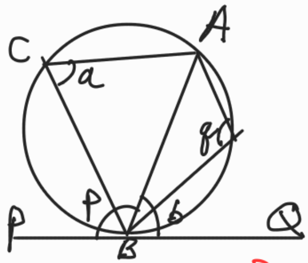
$c = 90^\circ - a$

$b + c = 90^\circ$

$b + 90^\circ - a = 90^\circ$

$\therefore \underline{a = b}$

proof II



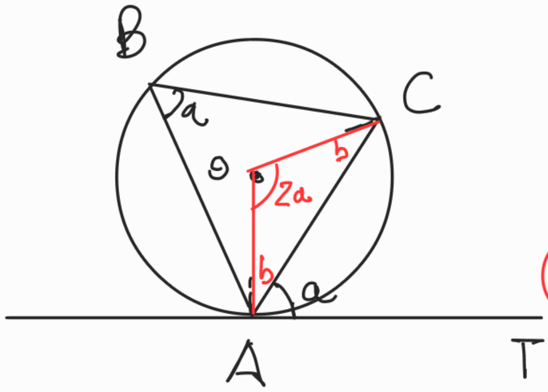
$p + b = 180^\circ$

$a + q = 180^\circ$

$a + q = p + b$

since $a = b$

$\therefore \underline{p = q}$



For $\triangle ABC$: $b + b + 2a = 180^\circ$

$2b + 2a = 180^\circ$

$b + a = 90^\circ$

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$\therefore$ If $\angle CBA = \angle CAT$,

Then TA is tangent of the circle

(Converse of ∠ in alt segment)

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