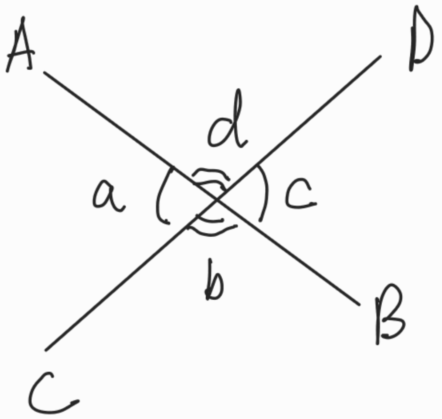
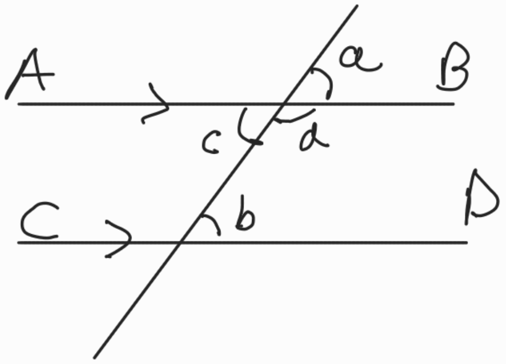
14 Basic Properties of Circles

Revision angles in straight and parallel lines



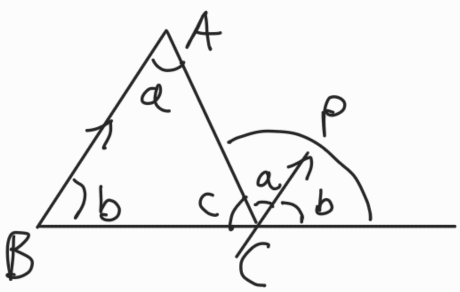
a) $a+b+c+d=360^\circ$ ($\angle$s at a point)
b) $a=c$, $b=d$ (vert opp $\angle$s)
c) $a+b=180^\circ$ (adj $\angle$s on a st line)



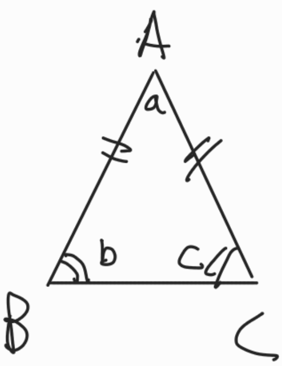
a) If $AB // CD$, then
(i) $a=b$ (corr $\angle$s $AB//CD$)
(ii) $b=c$ (alt $\angle$s $AB//CD$)
(iii) $b+d=180^\circ$ (int. $\angle$s $AB//CD$)

b) (i) If $a=b$, then $AB//CD$ (corr $\angle$s equal)
(ii) If $b=c$, then $AB//CD$ (alt $\angle$s equal)
(iii) If $b+d=180^\circ$ then $AB//CD$ (int $\angle$s supp)

Revision angles related to $\triangle$

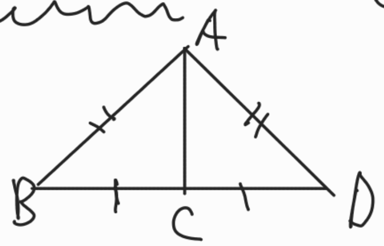


(a) $a+b+c=180^\circ$ ($\angle$ sum of $\triangle$)
(b) $p=a+b$ (ext $\angle$ of $\triangle$)



(c) (i) if $AB=AC$, then $b=c$ (base $\angle$s, isos $\triangle$)
(ii) if $b=c$, then $AB=BC$ (sides opp equal $\angle$s)

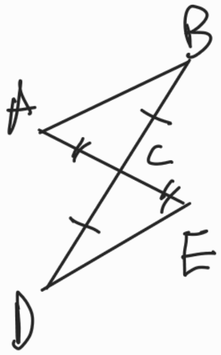
Revision Congruent Triangles (similar size, shape)



3 pairs of sides equal (SSS)

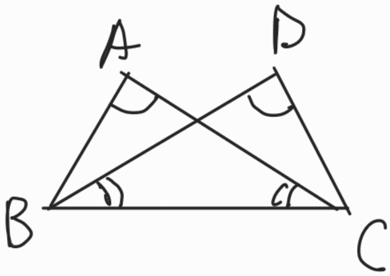
$AB = AD$ (given)
$BC = DC$ (given)
$AC = AC$ (common)
$\therefore \triangle ABC \cong ADC$ (SSS)

$\therefore$ ← therefore $\cong$ ← congruent



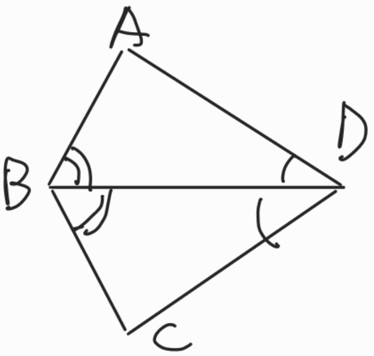
2 pairs of sides and one angle (SAS)

$AC = EC$ (given)
$BC = DC$ (given)
$\angle ACD = \angle ECD$ (vert opp $\angle$s)
$\therefore \triangle ABC \cong \triangle EDC$ (SAS)



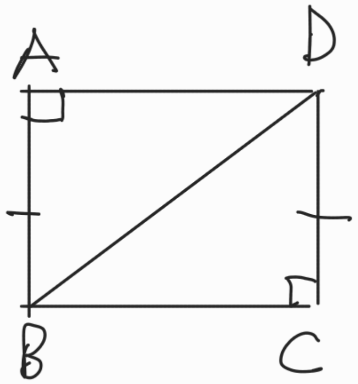
2 pairs of angles and 1 pair of sides (AAS)

$\angle BAC = \angle CDB$ (given)
$\angle ACB = \angle DBC$ (given)
$BC = CB$ (common)
$\therefore \triangle ABC \cong \triangle DCB$ (AAS)



2 pairs of angles and their included sides (ASA)

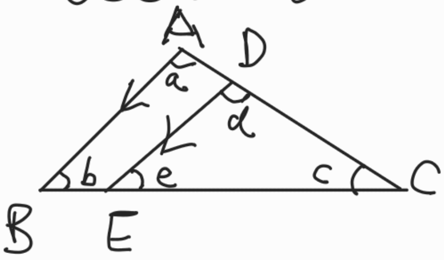
$\angle ABD = \angle CBD$ (given)
$\angle ADB = \angle CDB$ (given)
$BD = BD$ (common)
$\therefore \triangle ABD \cong CBD$ (ASA)



The hypotenuses and 1 pair of sides of $\llcorner$ $\triangle$ (RHS)

$\angle BAD = \angle DCB = 90°$ (given)
$AB = CD$ (given)
$BD = DB$ (common)
$\therefore \triangle ABC \cong \triangle CDB$ (RHS)

Revision Similar Triangles (similar shape, diff size)



$a = d$ (corr $\angle$s $AB \parallel DE$)

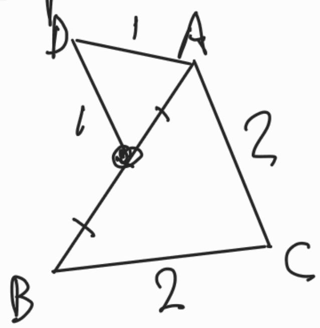
$b = e$ (corr $\angle$s $AB \parallel DE$)

$\angle ACB = \angle DCE$ (common $\angle$s)

$\therefore \triangle ABC \sim \triangle DEC$ (AAA)

3 pairs of angles are equal (AAA)

3 pairs of sides are proportional (3 sides proportional)



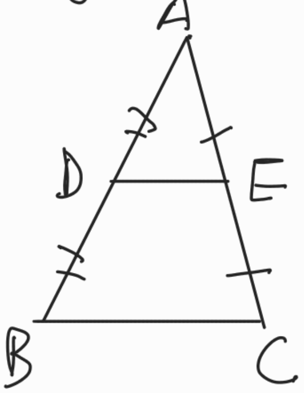
$AE / AB = 1/2$

$AD / AC = 1/2$

$DE / CB = 1/2$

$\therefore \triangle ABC \sim \triangle AED$ (3 sides proportional)

2 pairs of sides are proportional and their included angles are equal



$AD / AB = 1/2$

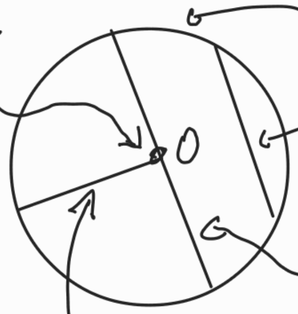
$AE / AC = 1/2$

$\angle DAE = \angle BAC$ (common $\angle$)

$\therefore \triangle ADE \sim \triangle ABC$ (ratio of 2 sides, incl $\angle$)

Finish Revision, Let's Start !

Chords of a Circle



centre of circle

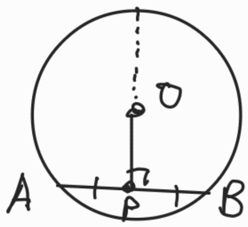
circumference

chord: both end points are on the circumference

diameter: a chord passing through O

radius: a line joining O and the circumference

A: Line from centre to a cord



① * If a line from O to a chord is ⊥, then it bisects the cord AP=PB

(line from centre ⊥ cord, bisects chord)

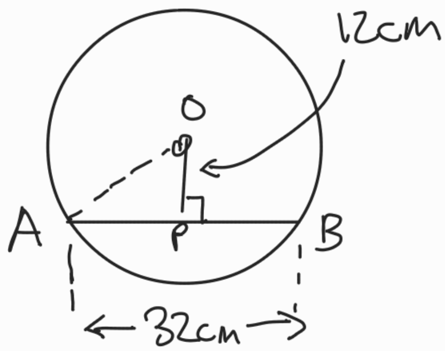
② * If a line from O meets the AB midpoint, then the line is ⊥ to chord

(Line from centre to midpoint of chord, ⊥ chord)

③ * The perpendicular bisector of chord, passes through O

(⊥ bisector of chord passes through centre)

Example



find the radius of circle

$\because OP \perp AB$ (given)

$\therefore AP = BP$ (line from centre ⊥ cord, bisects chord)

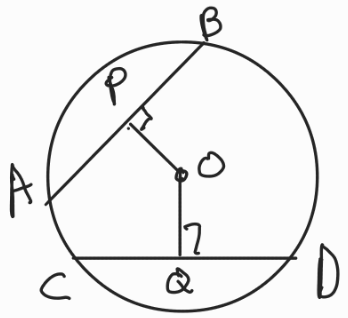
$= \frac{1}{2} \times 32 = 16\text{cm}$ //

$OA^2 = AP^2 + OP^2$

$OA = \sqrt{16^2 + 12^2} = 20\text{ cm}$ //

radius : 20 cm

B Distance between Chords and the Centre



4 * If 2 chords are equal length they are equal distance from centre

if $AB = CD$ then $OP = OQ$

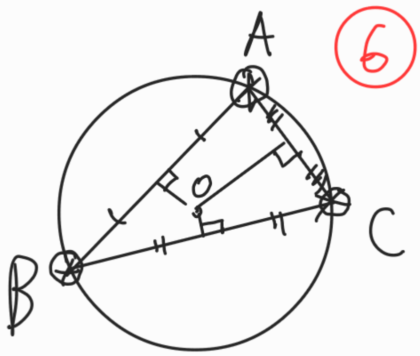
(equal chords, equal distance from O)

5 * If 2 chords are equal distance from O, then their lengths are equal

(chords equidistant from O, are equal)

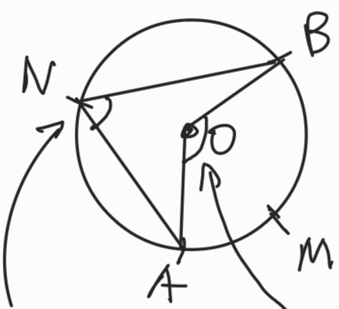
if $OP = OQ$ then $AB = CD$

C Circle Passing 3 Non-collinear Points



6 * There is only one circle, passing through any A, B, C (3 points)

Angles of a Circle



major arc, longer!

$\overset{\frown}{ANB}$: Arc joining A+B, passing through N

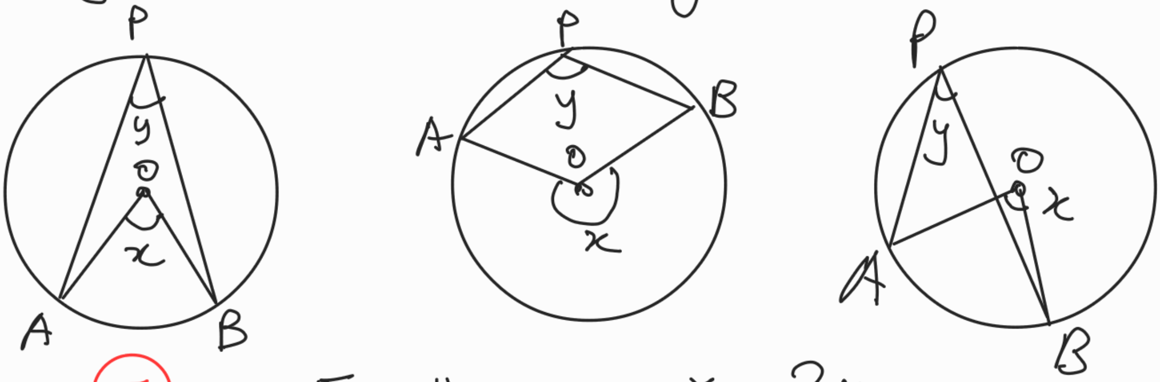
$\overset{\frown}{AMB}$: Arc joining A+B, passing through M

minor arc, shorter!

$\angle ANB$: angle at the circumference

$\angle AOB$: angle at the centre

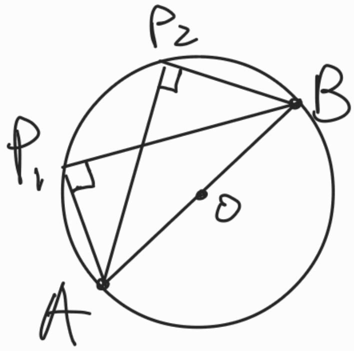
Angle at the centre and angle at the circumference



7 For all cases $x = 2y$

($\angle$ at O is $2\times\angle$ at $\odot^{ce}$)

Special case, when $x = 180^\circ$



8 * Any angle in a semicircle is 90°

If AB is the diameter, then $\angle APB = 90^\circ$

($\angle$ in semicircle)

9 * If $APB = 90^\circ$, then AB is the diameter

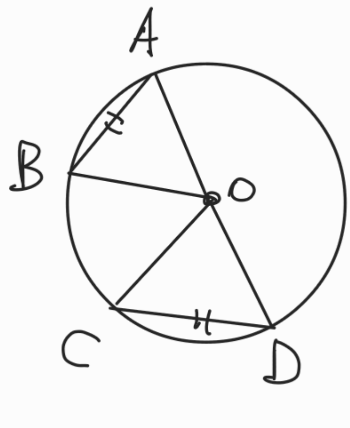
(Converse of $\angle$ in semicircle)

10 * Angles in the same segment of a circle are equal

$\angle AP_1B = \angle AP_2B$ (for any AB locations)

($\angle$s in the same segment)

Arcs, Chords and Angles of Circles



If $AB = CD$

Then $OA = OB = OC = OD$ (radius of $\bigcirc$)

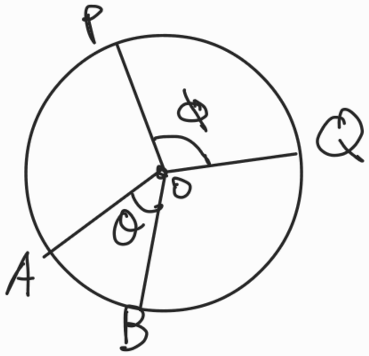
$\triangle AOB \cong \triangle COD$ (SSS)

11 * if $AB = CD$, then $\overset{\frown}{AB} = \overset{\frown}{CD}$ (equal chords, equal arcs)

12 * if $\overset{\frown}{AB} = \overset{\frown}{CD}$ then $AB = CD$ (equal arcs, equal chords)

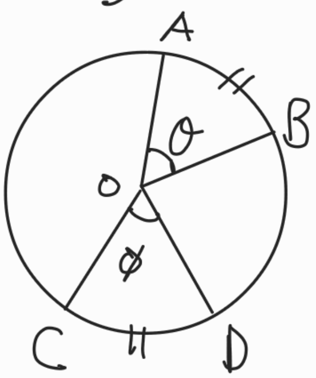
Equal Chords on Equal Arcs

Arcs, Angles at the Centre, Angles at the Circumference



13 * $\overset{\frown}{AB} : \overset{\frown}{PQ} = \theta : \phi$

(arc prop to $\angle$s at centre)

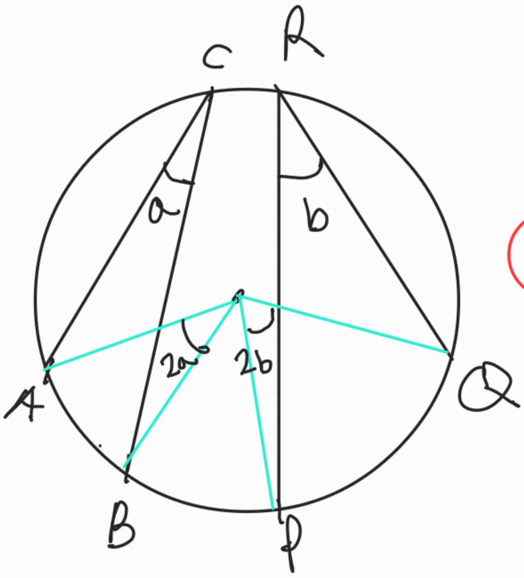


14 * if $\theta = \phi$, then $\overset{\frown}{AB} = \overset{\frown}{CD}$

(equal $\angle$s equal arcs)

15 * if $\overset{\frown}{AB} = \overset{\frown}{CD}$, then $\theta = \phi$

(equal arcs, equal $\angle$s)

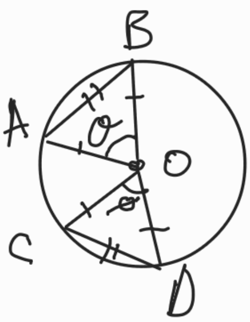


$\overset{\frown}{AB} : \overset{\frown}{PQ} = 2a : 2b$

$= a : b$

16 * $\overset{\frown}{AB} : \overset{\frown}{PQ} = a : b$

(arcs prop to $\angle$s at $\odot^{ce}$)



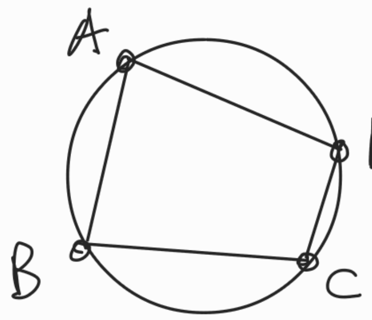
17 if $\theta = \phi$ then $AB = CD$

(equal $\angle$s, equal chords)

18 if $AB = CD$ then $\theta = \phi$

(equal chords, equal $\angle$s)

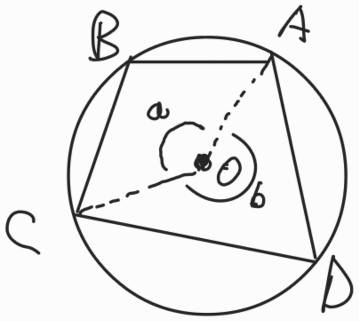
CYCLIC QUADRILATERALS (cyclic quad)



If quad-ri-lateral lies on the same circle:

ABCD is a Cyclic Quadrilateral

Opposite Angles



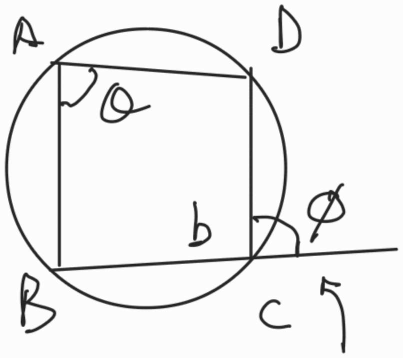
19 The opposite angles of a Cyclic Quad are supplementary (i.e. 180°)

$\angle ABC + \angle ADC = 180°$
$\angle BAD + \angle BCD = 180°$
(opp $\angle$s cyclic quad)

Proof:

$a = 2 \times \angle ABC$ ($\angle$ at centre, $2 \times \angle$ at $\odot^{ce}$)
$b = 2 \times \angle ADC$ ($\angle$ at centre, $2 \times \angle$ at $\odot^{ce}$)
$a + b = 360°$ ($\angle$s at a point)
$\therefore \quad 2 \times \angle ABC + 2 \times \angle ADC = 360°$
$\angle ABC + \angle ADC = 180°$

Exterior Angle

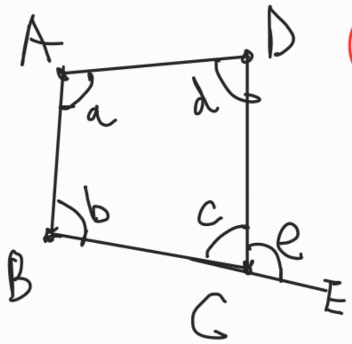


20 $\theta = \phi$
(ext $\angle$, cyclic quad)

proof $b + \theta = 180°$ (opp $\angle$ cyclic quad)
$b + \phi = 180°$ (adj $\angle$ on straight line)
$b + \theta = b + \phi$
$\theta = \phi$

Converse of Opposite Angles

ref 19



21 if $a + c = 180°$

or if $b + d = 180°$

then ABCD is a <u>cyclic quad</u>

A, B, C, D are <u>con cyclic</u>

(opp ∠s supp)

22

If BCE is a straight line, and $a = e$

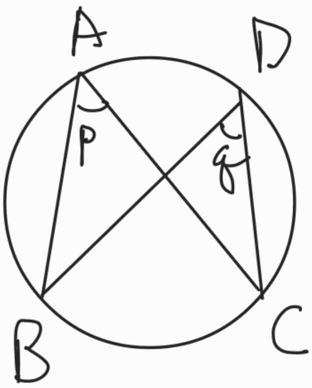
then A, B, C, D are concyclic

↳ in the same circle

(ext.∠ = int opp ∠)

Converse of Same Segment

ref 10



23 if $p = q$, then A, B, C, D

are con cyclic

(Converse of ∠s in same segment)

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