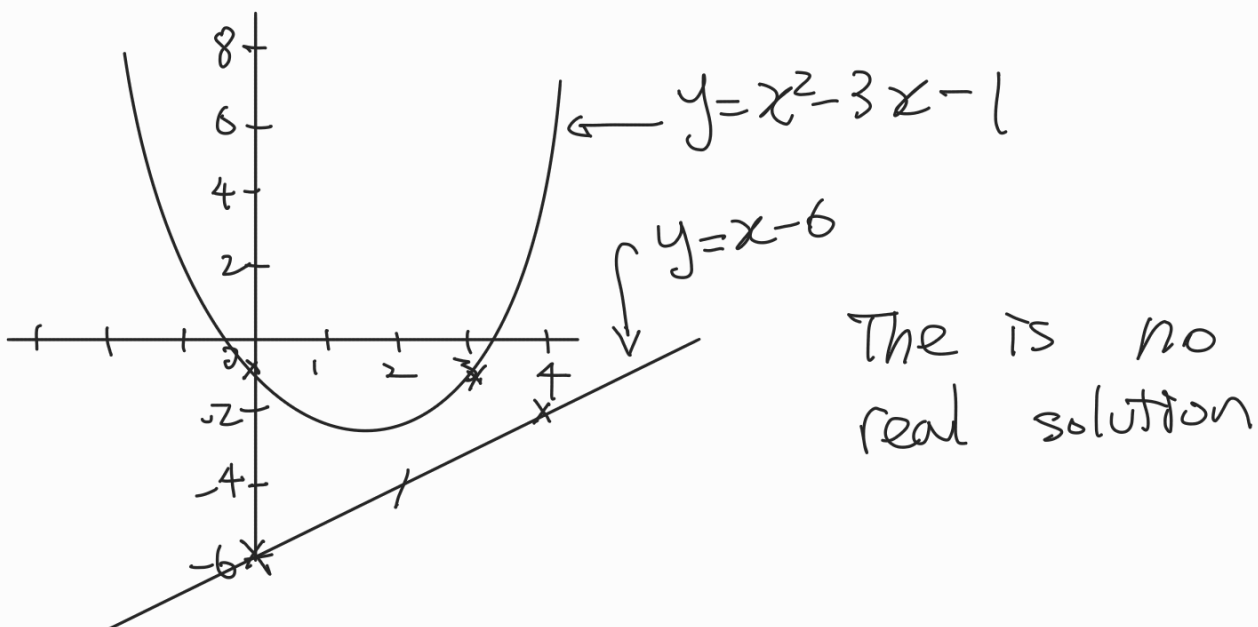
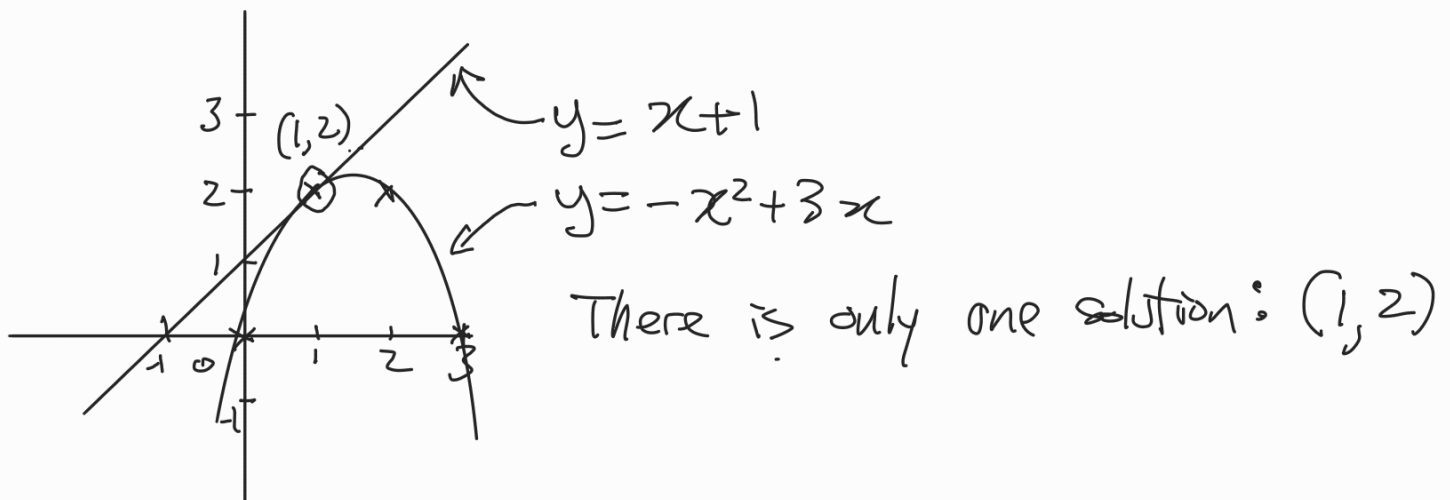
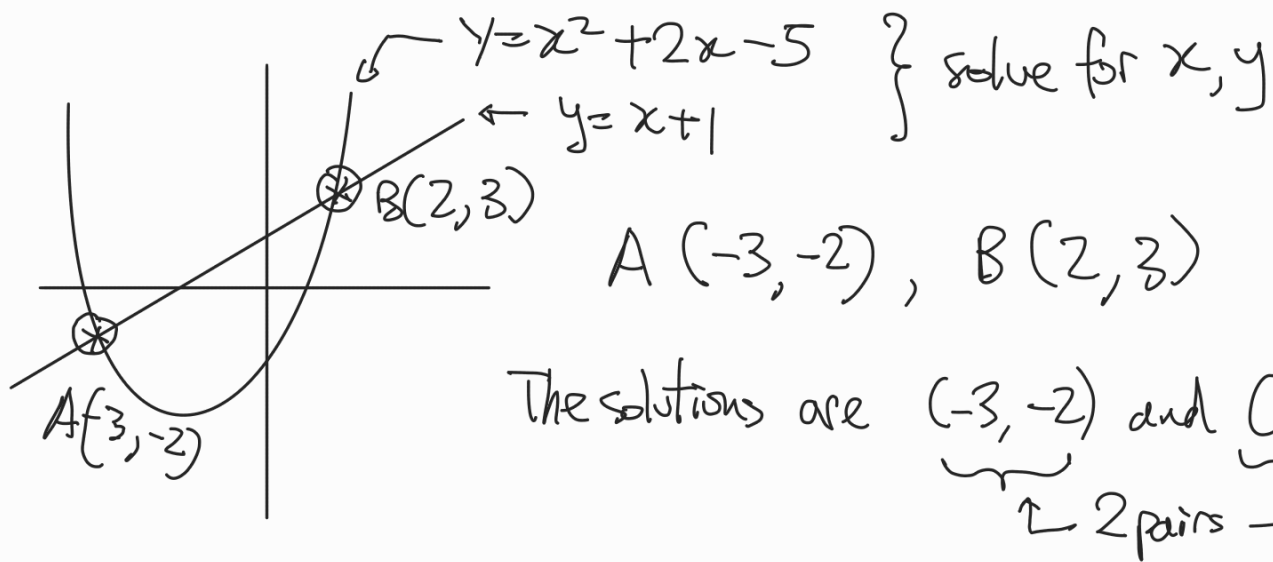


13 Simultaneous Equations

Solve by Graphical Method



Solving by Algebraic Method

Just use the technique of solving equations

Example $y = 3x - 2$ ---- (1) } solve for x, y
 $y = x^2$ ---- (2)

By sub (1) into (2)

$$3x - 2 = x^2$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0 \quad \underline{x = 1 \text{ or } 2}$$

By using $x = 1, 2$ and sub into (1)

$$x = 1 \Rightarrow y = 3(1) - 2 = 1$$

$$x = 2 \Rightarrow y = 3(2) - 2 = 4$$

Answer : $(x, y) = (1, 1) \text{ and } (2, 4)$ 2 pairs

Example $x + y = 4$ ---- (1)
 $3x^2 + y^2 = 12$ ---- (2)

From (1) : $y = -x + 4$ ---- (3)

Sub (3) into (2) : $3x^2 + (-x + 4)^2 = 12$

$$3x^2 + x^2 - 8x + 16 = 12$$

$$4x^2 - 8x + 4 = 0$$

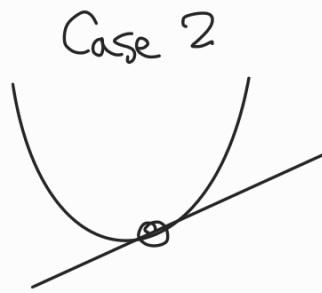
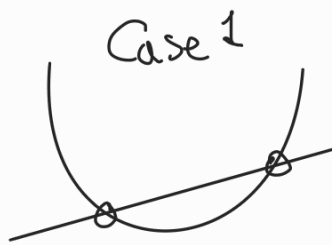
$$x^2 - 2x + 1 = 0$$

$$(x-1)(x-1) \quad x = 1$$

Sub 1 into (3) $y = -(1) + 4 = 3$

Answer : $(x, y) = (1, 3)$ one pair

Find out the number of solution pairs



$$\Delta = b^2 - 4ac \quad \text{for } ax^2 + bx + c = 0$$

Case 1: $\Delta > 0$

Case 2: $\Delta = 0$

Case 3: $\Delta < 0$

Example:
$$\left. \begin{array}{l} y = 7x \text{ --- (1)} \\ y = x^2 + 6 \text{ --- (2)} \end{array} \right\} \text{How many solution pairs?}$$

Sub (1) into (2) $7x = x^2 + 6$

$$\underbrace{1}_{a}x^2 - \underbrace{7}_{b}x + \underbrace{6}_{c} = 0$$

$$\Delta = (-7)^2 - 4 \times 1 \times 6$$
$$= 25 > 0$$

$\underbrace{\hspace{1.5cm}} \rightarrow 2 \text{ pairs of intersection}$

Solving Practical Problems Leading to Simultaneous Eqns

Step 1: Study and understand

Step 2: Set 2 variables (x, y) into 2 equations

Step 3: Solve the 2 simultaneous equations

Step 4: Check the solutions.

Example

The difference of 2 positive numbers is 3 and their product is 208. Find the 2 numbers

Solution

Let x, y be the 2 numbers

$$x - y = 3 \quad \text{--- (1)}$$

$$xy = 208 \quad \text{--- (2)}$$

$$\text{From (1)}: x = y + 3 \quad \text{--- (3)}$$

$$\text{Sub (3) into (2)}: (y + 3)y = 208$$

$$y^2 + 3y - 208 = 0$$

$$(y + 16)(y - 13) = 0$$

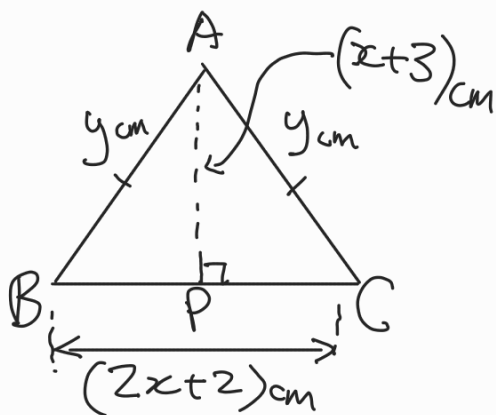
$$y = 13 \text{ and } -16 \rightarrow \text{rejected not positive}$$

$$\text{Sub } y = 13 \text{ into (3)}: x = 13 + 3 = 16$$

$$\text{Answer: } x, y \Rightarrow 13, 16$$

Example

An isosceles triangle $AB = AC = y$ and $BC = (2x + 2)\text{cm}$. AP Height is $(x + 3)\text{cm}$



(a) Show that $y^2 = 2x^2 + 8x + 10$

(b) If the perimeter of ABC is 32 cm find the values of x and y .

Solution:

(a) Consider $\triangle ABP$ and $\triangle ACP$

$AP = AP$; $AB = AC$, $\angle ABP = 90^\circ = \angle ACP$
Therefore $BP = CP$

$$BP = \frac{2x+2}{2} = (x+1) \text{ cm}$$

Consider $\triangle ABP$

$$AB^2 = AP^2 + BP^2$$

$$\begin{aligned} y^2 &= (x+3)^2 + (x+1)^2 \\ &= x^2 + 6x + 9 + x^2 + 2x + 1 \\ &= \underline{2x^2 + 8x + 10} \quad \text{----- (1)} \end{aligned}$$

(b) The perimeter of $\triangle ABC$ is 32 cm

$$y + y + 2x + 2 = 32$$

$$2y + 2x = 30$$

$$x + y = 15 \quad \text{----- (2)}$$

$$y = 15 - x \quad \text{----- (3)}$$

Sub (3) into (1)

$$(15-x)^2 = 2x^2 + 8x + 10$$

$$225 - 30x + x^2 = 2x^2 + 8x + 10$$

$$x^2 + 38x - 215 = 0$$

$$(x+43)(x-5) = 0$$

$$x = 5 \text{ or } -43 \rightarrow \text{rejected length cannot be negative}$$

Sub $x=5$ into (3)

$$y = 15 - 5 = 10$$

$$\underline{\text{ANSWER: } x, y \Rightarrow 5, 10}$$



