

12 More about Quadratic Equations (QE)

A Fractional Equations

Equations involving fractions e.g. $\frac{x-1}{3} = \frac{2}{x+4}$

$$\begin{aligned}\frac{x-1}{3} &= \frac{2}{x+4} \Rightarrow (x-1)(x+4) = 2 \times 3 \\ x^2 + 3x - 4 &= 6 \\ x^2 + 3x - 10 &= 0 \\ (x+5)(x-2) &= 0 \\ x &= \underline{\underline{-5 \text{ or } 2}}\end{aligned}$$

B Equations of Higher Degree

e.g. $x^4 - x^2 - 12 = 0 \Rightarrow$ Let $y = x^2$

$$\begin{aligned}y^2 - y - 12 &= 0 \\ (y-4)(y+3) &= 0 \\ y &= 4 \text{ or } -3\end{aligned}$$

Since $y = x^2 \Rightarrow x^2 = 4 \text{ or } -3$ (rejected)
 $\sqrt{-3}$ is not a real value

$$x = \underline{\underline{2 \text{ or } -2}}$$

e.g. $\sqrt{x+4} + x = 8 \Rightarrow$ Let $y = \sqrt{x+4}$

$$\begin{aligned}y^2 &= x+4 \\ x &= y^2 - 4\end{aligned}$$

Put back into eqn $\Rightarrow$

$$\begin{aligned}y + y^2 - 4 &= 8 \\ y^2 + y - 12 &= 0 \\ (y-3)(y+4) &= 0 \\ y &= -4 \text{ or } 3\end{aligned}$$

Since $y = \sqrt{x+4} \Rightarrow \sqrt{x+4} = 3$ or $\sqrt{x+4} = -4$ ~~rejected~~ $x = 3^2 - 4 = 5 //$

C Exponential Equations and Logarithmic Equations

Example 1: $2^{2x} - 2(2^x) - 8 = 0$

Let $y = 2^x \Rightarrow y^2 = (2^x)^2 = 2^{2x}$

$$y^2 - 2y - 8 = 0$$

$$(y-4)(y+2) = 0$$

$$y = 4 \text{ or } -2$$

Since $y = 2^x \Rightarrow$

$$2^x = 4$$

$$2^x = 2^2$$

$$x = 2 //$$

~~$2^x = -2$~~

rejected
 $2^x > 0$
for all real x

Example 2: $(\log x)^2 + \log x^2 - 3 = 0$

Let $y = \log x \Rightarrow y^2 + 2y - 3 = 0$

$$(y+3)(y-1) = 0$$

$$y = -3 \text{ or } 1$$

Since $y = \log x$

$$-3 = \log x \text{ or } 1 = \log x$$

$$x = 10^{-3}$$

$$= 0.001 //$$

$$x = 10^1$$

$$= 10 //$$

Problems that can be transformed into QE

- Step 1 : Study the problem. Draw a diagram if needed.
- Step 2 : Set the unknown variable. Relate all other variables to the unknown variable.
- Step 3 : Set up the equation
- Step 4 : Transform the equation into QE form, and solve
- Step 5 : Check the solutions, reject the non-answer.

Example 1 : The sum of the reciprocals of two consecutive integers is $25/156$. Find the integers.

Step 1, 2 : Think! 2 integers are $\frac{1}{n}$, $\frac{1}{n+1}$

Step 3 : $\frac{1}{n} + \frac{1}{n+1} = 25/156$

Step 4 : $\frac{(n+1) + n}{n(n+1)} = 25/156$

$$\Rightarrow \frac{2n+1}{n(n+1)} = 25/156$$

$$\Rightarrow 156(2n+1) = 25(n^2+n)$$

$$\Rightarrow 312n + 156 = 25n^2 + 25n$$

$$\Rightarrow 25n^2 - 287n - 156 = 0$$

$$\Rightarrow (n-12)(25n+13) = 0$$

Step 5 : $n = 12$ or $-13/25$ → rejected, not an integer

⇒ Answer : The 2 integers are 12, 13

Example 2 : Mr. X bought potatoes with \$80 this week last week, Mr. X found he could buy 2 kg more with \$80. If the price difference between this week and last week is \$2;

Find : a) Amount of potatoes Mr. X bought this week.
b) The price of potatoes this week.

Solution : Amount of potatoes bought this week : x kg

Price (per kg) this week : $\$80/x$

Price (per kg) last week : $\$80/(x+2)$

Price difference : $\frac{80}{x} - \frac{80}{(x+2)} = 2$

Solve eqn :
$$\frac{80(x+2) - 80(x)}{x(x+2)} = 2$$

$$80(x+2) - 80(x) = 2x(x+2)$$

$$x^2 + 2x - 80 = 0$$

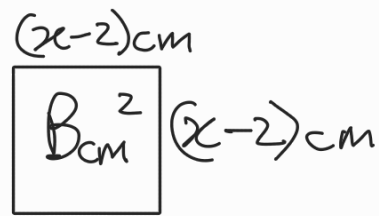
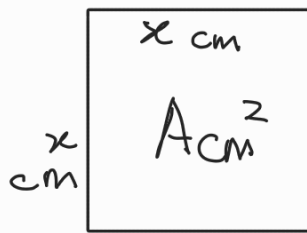
$$(x-8)(x+10) = 0$$

$$x = 8 \text{ or } -10 \rightarrow \text{rejected no negative amount}$$

Answer : a) 8 kg

$$b) \$80/8 = \$10 //$$

Example 3:



It is known that $A : B = 2x : (x+1)$

$$\Rightarrow \frac{x^2}{(x-2)^2} = \frac{2x}{x+1}$$

$$\Rightarrow \frac{x}{(x-2)^2} = \frac{2}{x+1}$$

$$\Rightarrow x^2 + x = 2(x^2 - 4x + 4)$$

$$x^2 - 9x + 8 = 0$$

$$(x-8)(x-1) = 0$$

$$x = 8 \text{ or } 1 \rightarrow \text{rejected } x-2 < 0$$

The 2 squares are $8 \text{ cm} \times 8 \text{ cm}$ and $6 \text{ cm} \times 6 \text{ cm}$ //

