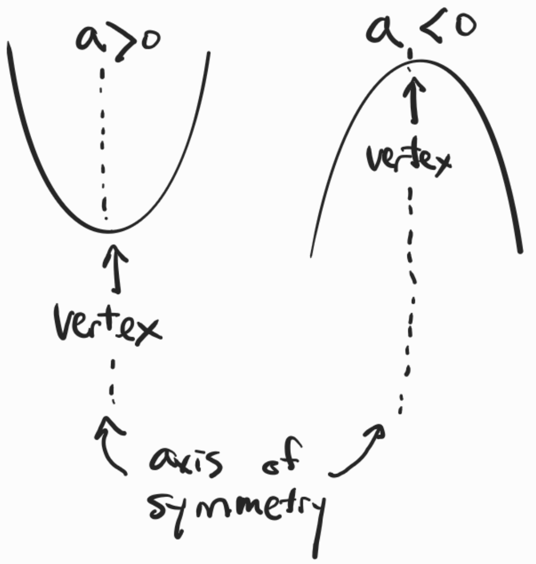
8 QUADRATIC FUNCTIONS AND GRAPHS

Review – graphs of QF

QF form $y = ax^2 + bx + c$, $a \neq 0$

Graphs:



[x-intercept

two intercept: if $\Delta = b^2 - 4ac > 0$

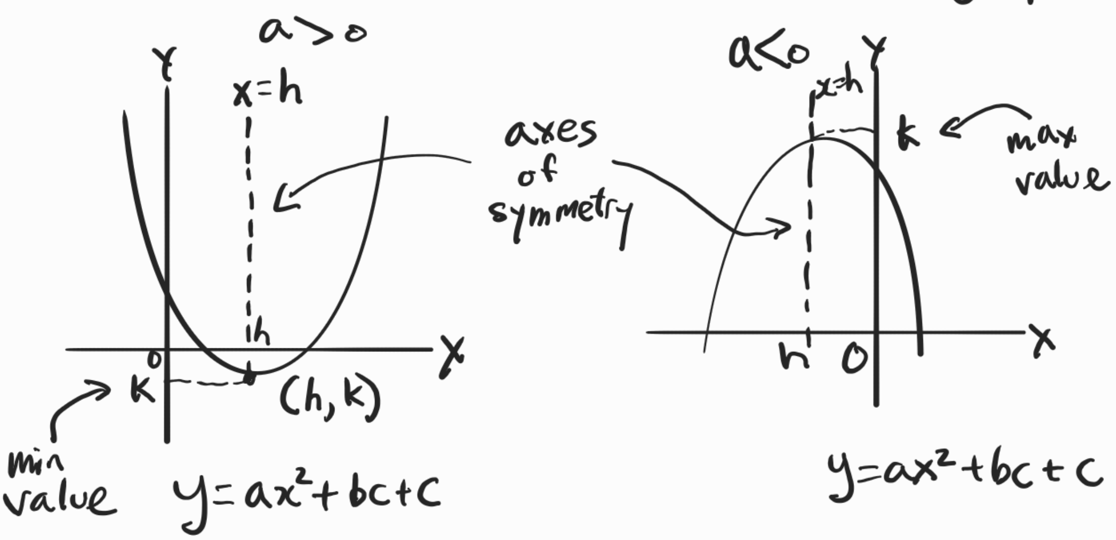
one intercept: if $\Delta = b^2 - 4ac = 0$

no intercept: if $\Delta = b^2 - 4ac < 0$

intercept is/are the roots of QE]

[y-intercept: c]

Find the max/min values of QF graphically



K: min/max value

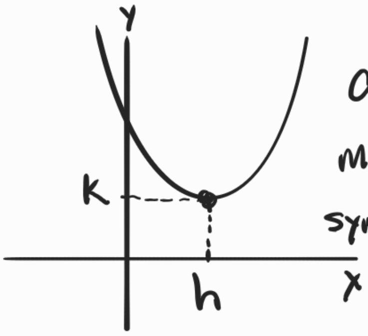
h: axis of symmetry

Example: $y = 3x^2 - 6x + 8$ has axis of symmetry $x = 1$

Find the minimum value of y.

Solution: axis of symmetry $x=1$ $\quad y = 3(1)^2 - 6(1) + 8 \quad y_{min} = 5$ //

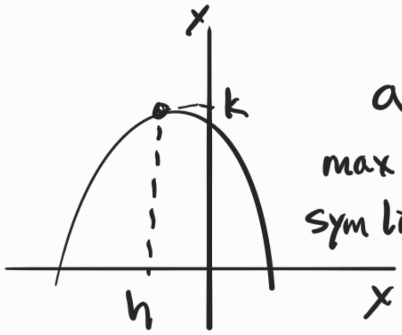
Optimum Values of QF form $y = a(x-h)^2 + k$



$a > 0$

min value : k

sym line : $x = h$



$a < 0$

max value : k

sym line : $x = h$

Optimum Values of QF form $y = ax^2 + bx + c$

A perfect square QF : $x^2 + 2px + p^2 = (x+p)^2$

$x^2 - 2px + p^2 = (x-p)^2$

example : $x^2 + 10x + 25 = x^2 + 2(x)(5) + 5^2 = (x+5)^2$

$x^2 - 8x + 16 = x^2 - 2(x)(4) + 4^2 = (x-4)^2$

Change $y = ax^2 + bx + c$ to $y = a\underline{(x-h)^2} + k$

obtain a perfect square term

Example : find the max value $y = x^2 - 2x + 5$

$y = -x^2 - 2x + 5$

$-(x^2 + 2x) + 5$ complete the square

$-(x^2 + 2x \underline{+1}) + 5 \underline{+1}$

$a=-1$ $\quad -(x+1)^2 + 6$ $\quad$ $h=-1$ $\quad$ $k=6$ $\quad$ max value $= 6$ //

$y = a(x-h)^2 + k$

Example : find the min value $y = 2x^2 - 9x + 13$

$y = 2\left(x^2 - \frac{9}{2}x\right) + 13$

$$y = 2\left[x^2 - \frac{9}{2}x + \left(\frac{9}{4}\right)^2 - \left(\frac{9}{4}\right)^2\right] + 13$$

$$= 2\left[\left(x - \frac{9}{4}\right)^2 - \frac{81}{16}\right] + 13$$

$$= 2\left(x - \frac{9}{4}\right)^2 - \frac{81}{8} + 13$$

$$= 2\left(x - \frac{9}{4}\right)^2 + \frac{23}{8}$$

min value $= 23/8$

$$y = a(x-h)^2 + k$$

Practical Problems

Example 1: John sells a toy at \$$x$.
The number of toys sold (y) is based on \$$x$. $y = -18x^2 + 468x$

a) Express y in standard form $y = a(x-h)^2 + k$

b) Find the max value of y, and its selling price \$$x$

a) Solution:

$$y = -18x^2 + 468x$$

$$= -18(x^2 - 26x)$$

$$= -18(x^3 - 26x + 13^2 - 13^2)$$

$$= -18[(x+13)^2 - 169]$$

$$= -18(x+13)^2 + 3042$$

b) max value: 3042 price: \$13

Example 2 : The cost of producing a pen is \$20. If the selling price is \$$x$, then the numbers sold is given by $8000-100x$.

a) Find the total profit \$$P$, expressed in terms of x.
b) Find the selling price for max profit.
c) Find the max profit and number of pens sold.

Solution :

a) profit for each pen $= \$(x-20)$
Total profit $P = (x-20)(8000-100x)$

b) For max profit, the selling price

$$
\begin{aligned}
P &= (x-20)(8000-100x) \\
&= 8000x - 160000 - 100x^2 + 2000x \\
&= -100x^2 + 10000x - 160000 \\
&= -100(x^2 - 100x) - 160000 \\
&= -100(x^2 - 100x - 50^2 + 50^2) - 160000 \\
&= -100[(x-50)^2 - 50^2] - 160000 \\
&= -100(x-50)^2 + 100(50^2) - 160000 \\
&= -100(x-\underline{50})^2 + 90000
\end{aligned}
$$

selling price = \$50

c) Max profit \$90,000

Number of pens sold $= 8000 - 100(50)$
$= 3000$

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