Review - Factorization (因數化) of Quadratic Polynomials (二次多項式)

Method 1: take out the common factor

e.g. $(x-2)^2 + (2x+3)(x-2)$

$= (x-2)[(x-2)+(2x+3)]$

$= (x-2)(x-2+2x+3)$

$= \underline{\underline{(x-2)(3x+1)}}$

Method 2: Use $a^2 - b^2 = (a+b)(a-b)$

e.g. $x^2 - 16 = x^2 - 4^2$

$= \underline{\underline{(x+4)(x-4)}}$

Method 3: Use $a^2 \pm 2ab + b^2 = (a \pm b)^2$

e.g. $x^2 - 6x + 9 = x^2 - (2)(3)x + (3)^2$

$= \underline{\underline{(x-3)^2}}$

Method 4: Use the cross inspection method

e.g. $x^2 - 3x + 2$

$\boxed{X}$

$$\begin{array}{ccccccc} -x & \text{or} & x & \times & -1 & \text{or} & 1 \\ -x & & x & & -2 & & 2 \\ \hline & & \checkmark & & \checkmark & & = -3x \end{array}$$

Ans: $\underline{\underline{(x-1)(x-2)}}$

Quadratic Equations and Roots (x^2 ← 二次)

general form : $ax^2 + bx + c = 0$
(a, b, c are constants, and $a \neq 0$)

e.g: $3\boxed{x^2} + x + 4$; $x^2 + 6$; $x^2 + 2x$
↳ this term must NOT be 0

Solving QE by factorization

e.g. $x^2 - 5x + 6 = 2$

$\Rightarrow x^2 - 5x + 4 = 0$ — step 1: standard form

$(x-4)(x-1) = 0$ — step 2: factorization

Let $(x-4) = 0 \Rightarrow (x-1) = 0$ — step 3: one factor be 0

$\underline{\underline{x = -1}}$

Let $(x-1) = 0 \Rightarrow (x-4) = 0$ — step 4: another factor 0

$\underline{\underline{x = 4}}$

Therefore $x = \underline{\underline{1 \text{ or } 4}}$ — Final answer

Solving QE by Quadratic Formula

(if you cannot factorize the equation)

QE formula: for $ax^2 + bx + c = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$(a \neq 0)$

Example 1 : (2 real roots)

$2x^2 - 11x + 14 = 0$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(14)}}{2(2)}$$

$$= \frac{11 \pm \sqrt{9}}{4} = \underline{\underline{2 \text{ or } \frac{7}{2}}}$$

Example 2: (repeated roots)

$$25x^2 - 20x + 4 = 0$$

$$x = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(25)(4)}}{2(25)}$$

$$= \frac{20 \pm \sqrt{0}}{50} = \underline{\underline{\frac{2}{5}}}$$

Example 3: (complex roots)

$$3x^2 + 3x + 1 = 0$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(3)(1)}}{2(3)}$$

$$= \frac{3 \pm \sqrt{-3}}{6} = \frac{3 \pm \sqrt{3}\sqrt{-1}}{6}$$

$$= \frac{3 + \sqrt{3}\,i}{6} \text{ or } \frac{3 - \sqrt{3}\,i}{6}$$

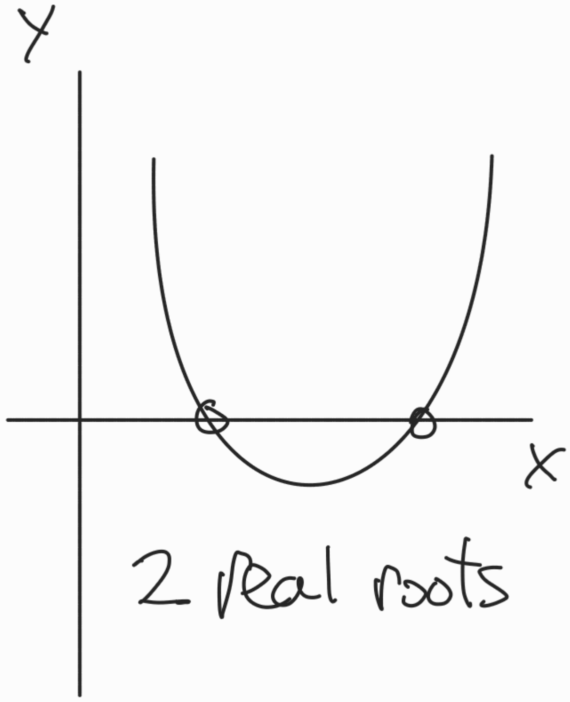
$$= \underline{\underline{\frac{1}{2} + i\frac{\sqrt{3}}{6} \text{ or } \frac{1}{2} - i\frac{\sqrt{3}}{6}}}$$

Solving by graphical method

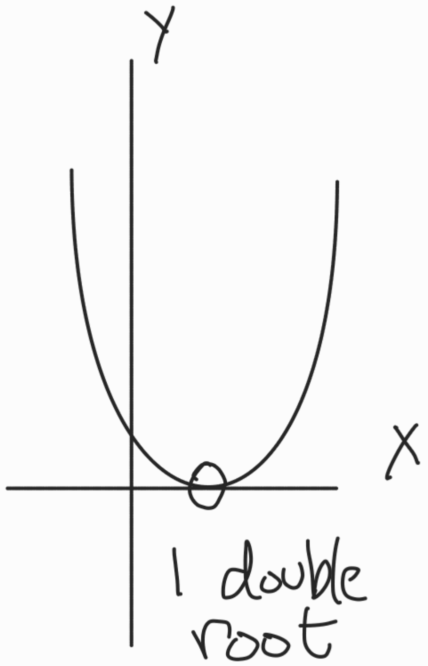
1st step: construct a table, find y from X

x	-2	-1	0	1	2	3	4	5
y								

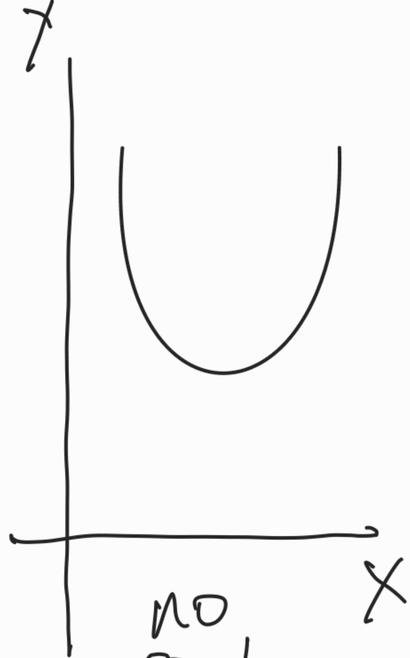
2nd step: plot the graph



2 real roots

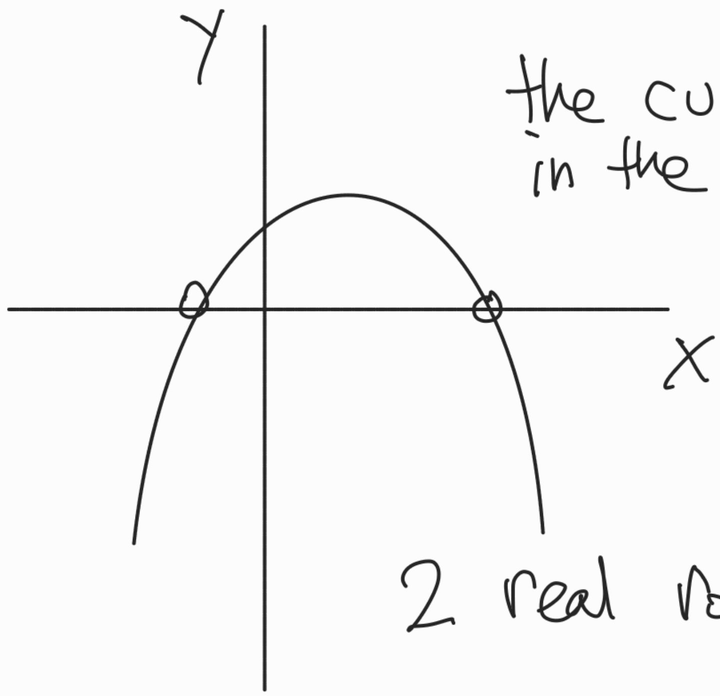


1 double root



no real root

For $-x^2$ plots



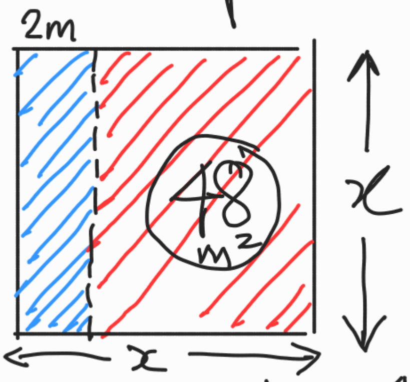
the curve bends in the negative direction

2 real roots

Practical Problems leading to QE

step 1 : Understand the problem
step 2 : Set a variable
step 3 : Set up the QE
step 4 : Solve the QE
step 5 : Check <u>all</u> solutions

Example 1: A square wall contains a blue strip and a red surface. Find x



area of red $= 48 m^2$
width of blue strip $= 2m$

1: understand the problem
2: $x \Rightarrow$ side of square wall
3: $x(x-2) = 48$
4: $x^2 - 2x - 48 = 0$
$(x-8)(x+6) = 0$
5: $x = 8$ or -6 (rejected)

ANSWER THE WALL IS $\underline{\underline{8m \times 8m}}$

Example 2: Product of two numbers is 720
Difference of two numbers is 29
The 2 numbers are positive values. Find two nos.

Solution: Two numbers are (x), $(x+29)$

$$x(x+29) = 720$$
$$x^2 + 29x - 720 = 0$$
$$(x+45)(x-16) = 0$$
$$x = -45 \text{ or } 16$$

negative, rejected

$$\boxed{\begin{aligned} x &= 16 \\ (x+29) &= 45 \end{aligned}}$$

— END —