Maths 01 - Real Numbers and Complex Numbers

REVIEW 1 basic operations of polynomials

e.g. 1 Simplify : $(+3x^2 - 2x + 3) - (+x^2 + x + 1)$

$= 3x^2 - 2x + 3 - x^2 - x - 1$ (remove brakets)

$= \underline{2x^2 - 3x + 2}$

e.g. 2 Multiply : $(+2x - 5) \times (+3x + 4)$

$= 6x^2 - 15x + 8x - 20$

$= \underline{6x^2 - 7x - 20}$

REVIEW 2 Surd is $\sqrt{p}$, where p : (i) integer, (ii) not square

for $a, b > 0$ $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

e.g. 1 Simplify : $\sqrt{20} + \sqrt{45}$

$= \sqrt{4}\sqrt{5} + \sqrt{9}\sqrt{5}$

$= 2\sqrt{5} + 3\sqrt{5}$

$= \underline{5\sqrt{5}}$

REVIEW 3 Solving Simultaneous Equations

e.g. 1 By Elimination :

$3x + y = 5$ ------ (1)

$+ \quad 4x - y = 2$ ---- (2)

$7x = 7$ ----- (1) + (2)

Therefore $\underline{x = 1}$ and $\underline{y = 2}$

e.g.2 By Substitution: $3x+y=5$ ....(1) ; $4x-y=2$ ...(2)

For (2): $4x-y=2$ $\quad 4x-2=y$

Sub into (1):

$$3x+[4x-2]=5$$

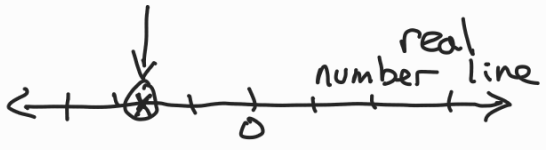
Therefore $\quad 7x=5+2 \quad x=1$

1.1 The Number System

REAL NUMBERS (實數)

- Rational Numbers (有理數)
  - Fractions (分數) $-\frac{2}{3}, \frac{1}{4}$
    - Terminating Decimals (終止小數) $2.5 = (2\frac{1}{2})$
    - Recurring Decimals (周期小數) $0.16 = (\frac{1}{6})$
  - Integars (整數) $-1, 0, 100$
    - Negative Integars (負數) $-4, -7$
    - Zero (零) 0
    - Positive Integars (正數) $+2, +20$
- Irrational Numbers (無理數) π, $\sqrt{3}$ ... etc

Properties of Real Numbers



1. Can be represented by points on the Real Number Line
2. Any real number $a^2 \geq 0$

COMPLEX NUMBER 複數

i is the complex no.

Definition: if $x^2=-1$, $x=\pm\sqrt{-1} = \pm i$, $i=\sqrt{-1}$

Therefore: $\sqrt{-a} = \sqrt{a} \times \sqrt{-1} = \sqrt{a} \times i$

complex numbers have 2 parts : $a + ib$

real 實數 ↑ a ; imaginary 虛數 ↑ b

e.g. $3 + i4$

$2 + i\sqrt{2}$

Operation of Complex Numbers

(+) and (−) : perform real and imaginary parts separately.

e.g. $(8-i) - (12-11i) = (8-12) + [-1-(-11)]i$

$= -4 + (-1+11)i$

$= \underline{\underline{-4 + 10i}}$

(×) : $z_1 \times z_2 = (a+bi)(c+di)$

$= ac + adi + bci + bdi^2$ (distributive law)

$= ac + (ad+bc)i + bd(-1)$

$= \underline{\underline{(ac-bd) + (ad+bc)i}}$

real ↑ imaginary ↑

e.g. : $(4-5i)(3+7i) = 12 + 28i - 15i - 35i^2$

$= 12 + 13i + 35$ $(i^2 = -1)$

$= \underline{\underline{47 + 13i}}$

real ↑ imaginary ↑

$(\frac{1}{i})$: eliminate any imaginary part in the denominator, by using $\underline{i \times i = -1}$

e.g. 1: $\frac{6}{i} = \frac{6}{i} \times \frac{i}{i} = \frac{6i}{i^2} = -6i$

e.g. 2: $\frac{3}{2+i} = \frac{3}{2+i} \times \frac{2-i}{2-i} = \frac{3(2-i)}{2^2 - i^2}$

$= \frac{6-3i}{2^2+1} = \frac{6-3i}{5} = \underbrace{\frac{6}{5} - \frac{3}{5}i}_{\text{(real) (imag)}}$

$(a+b)(a-b) = a^2 - b^2$

— END —