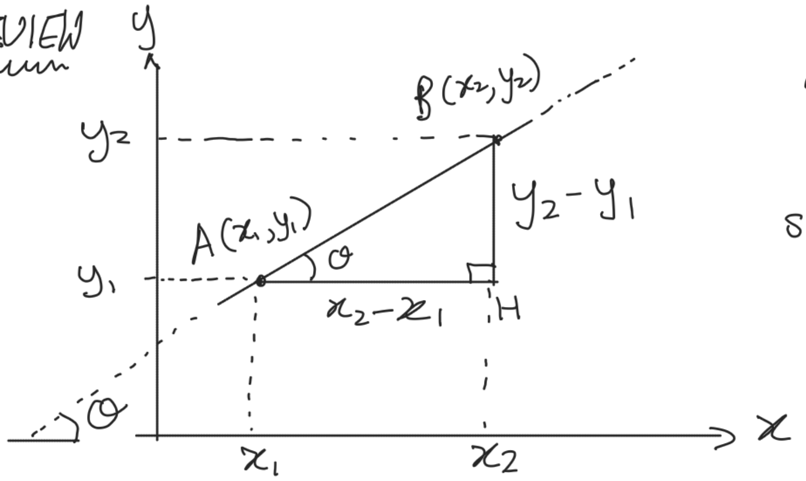
23 Circles and Locus

REVIEW



$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$$

Parallel Lines: If slopes of L_1, L_2 are m_1, m_2 respectively

If $m_1 = m_2$, then $L_1 // L_2$ (in parallel)

and vice versa

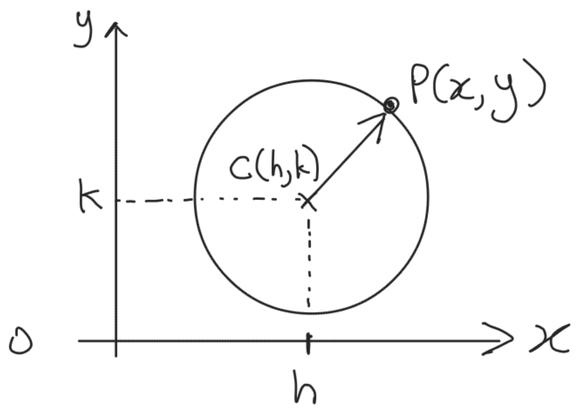
Perpendicular Lines: If $m_1 \times m_2 = -1$, then $L_1 \perp L_2$ (perpendicular)

and vice versa

Equation of a straight Line

$$\underbrace{y - y_1 = m(x - x_1)}_{m \text{ = slope}} \quad \text{or} \quad y = mx + c \;\; (c \text{ = y intercept})$$

Equations of Circles



CP = radius

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

or

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

standard form or centre radius form

Example: Find the equation of circle, with centre $(-1, 1)$ and passing through point $(-3, 2)$

Solution: Centre $(-1, 1)$

Radius $= \sqrt{[3-(-1)]^2 + (-2-1)^2} = 5$

Equation $= [x-(-1)]^2 + (y-1)^2 = 5^2$

$(x+1)^2 + (y-1)^2 = 25$

General Form

expand $(x-h)^2 + (y-k^2) = r^2$

$$x^2 \underbrace{-2hx}_{D} \underbrace{-2ky}_{E} + \underbrace{(h^2+k^2-r^2)}_{F} = 0$$

let $D = -2h$, $E = -2k$, $F = h^2+k^2-r^2$

so $$\boxed{x^2 + y^2 + Dx + Ey + F = 0}$$

features: x^2, y^2 have coefficients of 1

degree of equation $= 2$

D, E, F can be any real numbers

R.H.S. $= 0$, no xy term

Example: find the equation of a circle with centre $(-2, 3)$ and radius $\sqrt{5}$

Solution: $[x-(-2)]^2 + (y-3)^2 = (\sqrt{5})^2$

$x^2 + 4x + 4 + y^2 - 6y + 9 = 5$

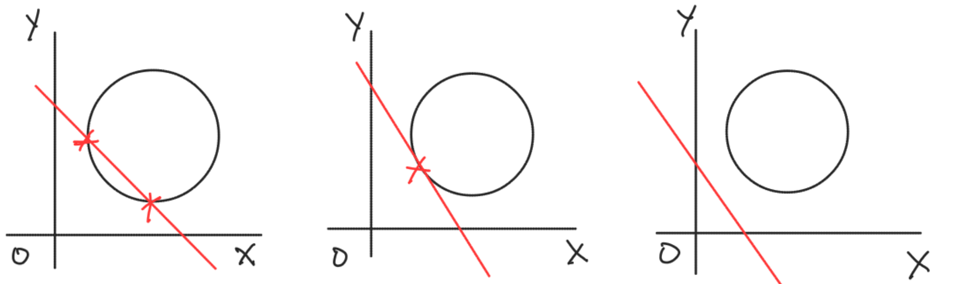
$x^2 + y^2 + 4x - 6y + 8 = 0$

Features of Equations of Circles

$$\boxed{\text{centre} = \left[-\frac{D}{2}, -\frac{E}{2}\right] \qquad \text{radius} = \sqrt{\underbrace{\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - F}}}$$

>0, real circle
$=0$, point circle
<0, imaginary circle

Intersection of a straight line and a circle



2 intersects   1 intersect   no intersect

Equation of circle: $x^2 + y^2 + Dx + Ey + F = 0$
Equation of a straight line: $y = mx + c$

Eliminate x or y, through substitution

e.g. $\quad \underline{(1+m^2)}\, x^2 + \underline{(2mc + D + Em)}\, x + \underline{(c^2 + Ec + F)} = 0$

$$A x^2 + B x + C = 0$$

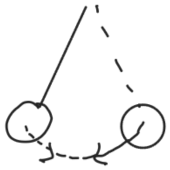
To find $x \Rightarrow$ $\boxed{x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A} \qquad \Delta = B^2 - 4AC}$

determinant Δ

If $\Delta > 0 \Rightarrow$ 2 points of intersection
$\Delta = 0 \Rightarrow$ 1 point of intersection
$\Delta < 0 \Rightarrow$ no point of intersection

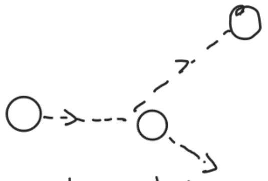
Concept of Locus (軌跡)

e.g.



pendulum

e.g.



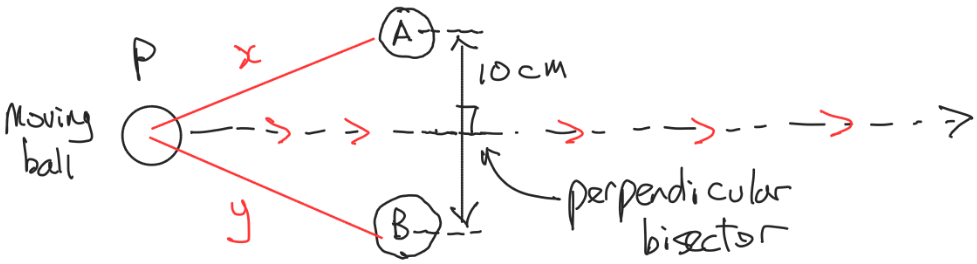
snooker ball

A Locus is a set of all points that satisfy a given specified condition

A loci of moving points can be expressed in algebraic equation

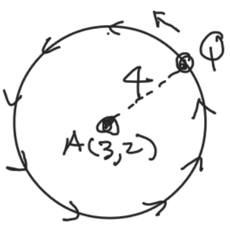
Examples

1. A moving point P is equi-distant from A and B which are 10 cm apart. The locus of P is the perpendicular bisector of AB



equi-distant $x = y$

2. Q is moving such that it is alaways 4 units from A(3,2). The equation of locus Q is :



$$\sqrt{(x-3)^2+(y-2)^2} = 4$$

$$x^2 + y^2 - 6x - 4y - 3 = 0$$

完