Learning Content of the Compulsory Part

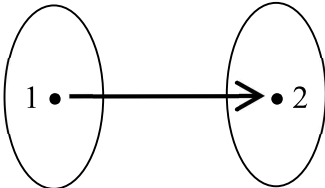## Notes:

1. *Learning units are grouped under three strands ("Number and Algebra", "Measures, Shape and Space" and "Data Handling") and a Further Learning Unit.*

2. *Related learning objectives are grouped under the same learning unit.*

3. *The <u>learning objectives underlined</u> are the Non-foundation Topics.*

4. *The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.*

5. *To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.*

6. *Schools may allocate up to 313 hours (i.e. 12.5% of the total lesson time) to those students who need more time for learning.*

Learning Unit	Learning Objective	Time	Remarks
Number and Algebra Strand			
1. Quadratic equations in one unknown	1.1 solve quadratic equations by the factor method	19	
	1.2 form quadratic equations from given roots		The given roots are confined to real numbers.
	1.3 solve the equation $ax^2 + bx + c = 0$ by plotting the graph of the parabola $y = ax^2 + bx + c$ and reading the x-intercepts		

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	1.4 solve quadratic equations by the quadratic formula		The following are **not** required for students taking only the Foundation Topics: • expressing nonreal roots in the form $a \pm bi$ • simplifying expressions involving surds such as $2 \pm \sqrt{48}$
	1.5 understand the relations between the discriminant of a quadratic equation and the nature of its roots		When $\Delta < 0$, students have to point out that "the equation has **no real roots**" or "the equation has **two nonreal roots**" as they are expected to recognise the existence of complex numbers in Learning Objective 1.8.
	1.6 solve problems involving quadratic equations		Teachers should select the problems related to students' experiences. Problems involving complicated equations such as $\dfrac{6}{x} + \dfrac{6}{x-1} = 5$ are required only in the Non-foundation Topics and dealt with in Learning Objective 5.4.

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	1.7 understand the relations between the roots and coefficients and form quadratic equations using these relations		The relations between the roots and coefficients include: • $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$, where α and β are the roots of the equation $ax^2 + bx + c = 0$ and $a \neq 0$.
	1.8 appreciate the development of the number systems including the system of complex numbers		The topics such as the hierarchy of the number systems and the conversion between recurring decimals and fractions may be discussed.
	1.9 perform addition, subtraction, multiplication and division of complex numbers		Complex numbers are confined to the form $a \pm bi$. Note: The coefficients of quadratic equations are confined to real numbers.
2. Functions and graphs	2.1 recognise the intuitive concepts of functions, domains and co-domains, independent and dependent variables	10	

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	2.2 recognise the notation of functions and use tabular, algebraic and graphical methods to represent functions		Representations like can also be accepted.
	2.3 understand the features of the graphs of quadratic functions		The features of the graphs of quadratic functions include: • the vertex • the axis of symmetry • the direction of opening • relations with the axes Students are required to find the maximum and minimum values of quadratic functions by the graphical method.
	2.4 <u>find the maximum and minimum values of quadratic functions by the algebraic method</u>		The method of completing the square is required. Students are required to solve problems related to maximum and minimum values of quadratic functions.

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3. Exponential and logarithmic functions	3.1 <u>understand the definitions of rational indices</u> 3.2 <u>understand the laws of rational indices</u> 3.3 <u>understand the definition and properties of logarithms (including the change of base)</u>	16	The definitions include $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$. The laws of rational indices include: • $a^p\, a^q = a^{p+q}$ • $\dfrac{a^p}{a^q} = a^{p-q}$ • $(a^p)^q = a^{pq}$ • $a^p\, b^p = (ab)^p$ • $\dfrac{a^p}{b^p} = \left(\dfrac{a}{b}\right)^p$ The properties of logarithms include: • $\log_a 1 = 0$ • $\log_a a = 1$ • $\log_a MN = \log_a M + \log_a N$ • $\log_a \dfrac{M}{N} = \log_a M - \log_a N$ • $\log_a M^k = k \log_a M$ • $\log_b N = \dfrac{\log_a N}{\log_a b}$

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	3.4 <u>understand the properties of exponential functions and logarithmic functions and recognise the features of their graphs</u>		The properties and features include: • the domains of the functions • the function $f(x) = a^x$ and $f(x) = \log_a x$ increases (decreases) as x increases for $a > 1$ $(0 < a < 1)$ • $y = a^x$ is symmetric to $y = \log_a x$ about $y = x$ • the intercepts with the axes • the rate of increasing/the rate of decreasing of the functions (by direct inspection)
	3.5 <u>solve exponential equations and logarithmic equations</u>		Equations which can be transformed into quadratic equations such as $4^x - 3 \cdot 2^x - 4 = 0$ or $\log(x - 22) + \log(x + 26) = 2$ are dealt with in Learning Objective 5.3.
	3.6 <u>appreciate the applications of logarithms in real-life situations</u>		The applications such as measuring earthquake intensity in the Richter Scale and sound intensity level in decibels may be discussed.

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	3.7 appreciate the development of the concepts of logarithms		The topics such as the historical development of the concepts of logarithms and its applications to the design of some past calculation tools such as slide rules and the logarithmic table may be discussed.
4. More about polynomials	4.1 perform division of polynomials	14	Methods other than long division are also accepted.
	4.2 understand the remainder theorem		
	4.3 understand the factor theorem		Students are required to use factor theorem to factorise polynomials such as $x^3 \pm a^3$.
	4.4 understand the concepts of the greatest common divisor and the least common multiple of polynomials		The terms "H.C.F." , "gcd", etc. can be used.
	4.5 perform addition, subtraction, multiplication and division of rational functions		Computation of rational functions with more than two variables is **not** required. Rational functions refer to algebraic fractions at Key Stage 3.
5. More about equations	5.1 use the graphical method to solve simultaneous equations in two unknowns, one linear and one quadratic in the form $y = ax^2 + bx + c$	10	

Learning Unit	Learning Objective	Time	Remarks
	5.2 <u>use the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic</u>		
	5.3 <u>solve equations (including fractional equations, exponential equations, logarithmic equations and trigonometric equations) which can be transformed into quadratic equations</u>		Solutions for trigonometric equations are confined to the interval from 0° to 360° .
	5.4 <u>solve problems involving equations which can be transformed into quadratic equations</u>		Teachers should select the problems related to students' experience.
6. Variations	6.1 understand direct variations and inverse variations, and their applications to solving real-life problems	7	
	6.2 understand the graphs of direct and inverse variations		
	6.3 understand joint and partial variations, and their applications to solving real-life problems		
7. Arithmetic and geometric sequences and their summations	7.1 <u>understand the concept and the properties of arithmetic sequences</u>	17	The properties of arithmetic sequences include: • $T_n = \frac{1}{2}\,(\,T_{n-1} + T_{n+1}\,)$ • if T_1 , T_2 , T_3 , … is an arithmetic sequence, then $k\,T_1 + a$, $k\,T_2 + a$, $k\,T_3 + a$, … is also an arithmetic sequence

Learning Unit	Learning Objective	Time	Remarks
	7.2 understand the general term of an arithmetic sequence		
	7.3 understand the concept and the properties of geometric sequences		The properties of geometric sequences include: • $T_n{}^2 = T_{n-1} \times T_{n+1}$ • if T_1 , T_2 , T_3 , … is a geometric sequence, then $k\,T_1$, $k\,T_2$, $k\,T_3$, … is also a geometric sequence
	7.4 understand the general term of a geometric sequence		
	7.5 understand the general formulae of the sum to a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems		Example: geometrical problems involving the sum of arithmetic or geometric sequences.
	7.6 explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems		Example: geometrical problems involving infinite sum of the geometric sequences.
	7.7 solve related real-life problems		Examples: problems about interest, growth or depreciation.

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8. Inequalities and linear programming	8.1 solve compound linear inequalities in one unknown	16	Compound inequalities involving logical connectives "and" or "or" are required. Solving the problems on triangle inequalities is required.
	8.2 solve quadratic inequalities in one unknown by the graphical method		
	8.3 solve quadratic inequalities in one unknown by the algebraic method		
	8.4 represent the graphs of linear inequalities in two unknowns in the rectangular coordinate plane		
	8.5 solve systems of linear inequalities in two unknowns		
	8.6 solve linear programming problems		
9. More about graphs of functions	9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions	11	Comparison includes domains, existence of maximum or minimum values, symmetry and periodicity.
	9.2 solve the equation $f(x) = k$ using the graph of $y = f(x)$		

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	9.3 solve the inequalities $f(x) > k$, $f(x) < k$, $f(x) \geq k$ and $f(x) \leq k$ using the graph of $y = f(x)$ 9.4 <u>understand the transformations of the function $f(x)$ including $f(x) + k$, $f(x+k)$, $k\,f(x)$ and $f(kx)$ from tabular, symbolic and graphical perspectives</u>		
Measures, Shape and Space Strand			
10. Equations of straight lines	10.1 understand the equation of a straight line	7	Students are required to find the equation of a straight line from given conditions such as: • the coordinates of any two points on the straight line • the slope of the straight line and the coordinates of a point on it • the slope and the y-intercept of the straight line Students are required to describe the features of a straight line from its equation. The features include: • the slope

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	10.2 understand the possible intersection of two straight lines		• the intercepts with the axes • whether it passes through a given point The normal form is **not** required. Students are required to recognise the relation between slope and inclination. Students are required to determine the number of intersection points of two straight lines from their equations. At Key Stage 3, students are required to solve simultaneous linear equations in two unknowns. Note: Teachers are suggested to arrange the teaching of this Learning Unit in the first term of S4.
11. Basic properties of circles	11.1 understand the properties of chords and arcs of a circle	23	The properties of chords and arcs of a circle include: • the chords of equal arcs are equal • equal chords cut off equal arcs • the perpendicular from the centre to a chord bisects the chord

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			• the straight line joining the centre and the mid-point of a chord which is not a diameter is perpendicular to the chord • the perpendicular bisector of a chord passes through the centre • equal chords are equidistant from the centre • chords equidistant from the centre are equal Students are required to understand that there is one and only one circle passing through given three non-collinear points. The property that the arcs are proportional to their corresponding angles at the centre should be discussed at Key Stage 3 when the formula for calculating arc lengths is being explicated.
	11.2 understand the angle properties of a circle		The angle properties of a circle include: • the angle subtended by an arc of a circle at the centre is double the angle subtended by the arc at any point on the remaining part of the circumference • angles in the same segment are equal • the arcs are proportional to their corresponding angles at the circumference

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			• the angle in a semi-circle is a right angle
			• if the angle at the circumference is a right angle, then the chord that subtends the angle is a diameter
	11.3 understand the properties of a cyclic quadrilateral		The properties of a cyclic quadrilateral include:
			• the opposite angles of a cyclic quadrilateral are supplementary
			• an exterior angle of a cyclic quadrilateral equals its interior opposite angle
	11.4 <u>understand the tests for concyclic points and cyclic quadrilaterals</u>		The tests for concyclic points and cyclic quadrilaterals include:
			• if A and D are two points on the same side of the line BC and $\angle BAC = \angle BDC$, then A, B, C and D are concyclic
			• if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic
			• if the exterior angle of a quadrilateral equals its interior opposite angle, then the quadrilateral is cyclic

Learning Unit	Learning Objective	Time	Remarks

	11.5 understand the properties of tangents to a circle and angles in the alternate segments		The properties include:
			
			• a tangent to a circle is perpendicular to the radius through the point of contact
			
			• the straight line perpendicular to a radius of a circle at its external extremity is a tangent to the circle
			
			• the perpendicular to a tangent at its point of contact passes through the centre of the circle
			
			• if two tangents are drawn to a circle from an external point, then:
			
			   − the distances from the external point to the points of contact are equal
			
			   − the tangents subtend equal angles at the centre
			
			   − the straight line joining the centre to the external point bisects the angle between the tangents
			
			• if a straight line is tangent to a circle, then the tangent-chord angle is equal to the angle in the alternate segment

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			• if a straight line passes through an end point of a chord of a circle so that the angle it makes with the chord is equal to the angle in the alternate segment, then the straight line touches the circle
	11.6 use the basic properties of circles to perform simple geometric proofs		Knowledge on geometry learnt at Key Stage 3 can be involved in the geometric proofs.
12. Loci	12.1 understand the concept of loci	6	
	12.2 describe and sketch the locus of points satisfying given conditions		The conditions include: • maintaining a fixed distance from a fixed point • maintaining an equal distance from two given points • maintaining a fixed distance from a line • maintaining an equal distance from two parallel lines • maintaining an equal distance from two intersecting lines

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	12.3 describe the locus of points with algebraic equations		Students are required to find the equations of simple loci, which include equations of straight lines, circles and parabolas (in the form of $y = ax^2 + bx + c$).
13. Equations of circles	13.1 understand the equation of a circle	7	Students are required to find the equation of a circle from given conditions such as: • the coordinates of the centre and the radius of the circle • the coordinates of any three points on the circle Students are required to describe the features of a circle from its equation. The features include: • the centre • the radius • whether a given point lies inside, outside or on the circle
	13.2 find the coordinates of the intersections of a straight line and a circle and understand the possible intersection of a straight line and a circle		Students are required to find the equations of tangents to a circle.

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14. More about trigonometry	14.1 understand the functions sine, cosine and tangent, and their graphs and properties, including maximum and minimum values and periodicity	25	Simplification of expressions involving sine, cosine and tangent of $-\theta$, $90° \pm \theta$, $180° \pm \theta$, $\dots$, etc. is required.
	14.2 solve the trigonometric equations $a \sin \theta = b$, $a \cos \theta = b$, $a \tan \theta = b$ (solutions in the interval from $0°$ to $360°$) <u>and other trigonometric equations (solutions in the interval from $0°$ to $360°$)</u>		Equations that can be transformed into quadratic equations are required only in the Non-foundation Topics and dealt with in Learning Objective 5.3.
	14.3 <u>understand the formula $\frac{1}{2} ab \sin C$ for areas of triangles</u>		
	14.4 <u>understand the sine and cosine formulae</u>		
	14.5 <u>understand Heron's formula</u>		
	14.6 <u>understand the concept of projection</u>		
	14.7 <u>understand the angle between a line and a plane, and the angle between 2 planes</u>		The concept of inclination is required.
	14.8 <u>understand the theorem of three perpendiculars</u>		

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	14.9 solve related 2-dimensional and 3-dimensional problems		3-dimensional problems include finding the angle between two lines, the angle between a line and a plane, the angle between two planes, the distance between points, the distance between a point and a line, and the distance between a point and a plane.
Data Handling Strand			
15. Permutations and combinations	15.1 understand the addition rule and multiplication rule in the counting principle	11	
	15.2 understand the concept and notation of permutation		Notations such as " P_r^n " , " $_nP_r$ " , " ^nP_r " , etc. can be used.
	15.3 solve problems on the permutation of distinct objects without repetition		Problems such as "permutation of objects in which three particular objects are put next to each other" are required. Circular permutation is **not** required.
	15.4 understand the concept and notation of combination		Notations such as " C_r^n " , " $_nC_r$ " , " ^nC_r " , " $\binom{n}{r}$ " , etc. can be used.

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	15.5 solve problems on the combination of distinct objects without repetition		
16. More about probability	16.1 recognise the notation of set language including union, intersection and complement	10	The concept of Venn Diagram is required.
	16.2 understand the addition law of probability and the concepts of mutually exclusive events and complementary events		The addition law of probability refers to "$P(A \cup B) = P(A) + P(B) - P(A \cap B)$".
	16.3 understand the multiplication law of probability and the concept of independent events		The multiplication law of probability refers to "$P(A \cap B) = P(A) \times P(B)$, where A and B are independent events".
	16.4 recognise the concept and notation of conditional probability		The rule "$P(A \cap B) = P(A) \times P(B \mid A)$" is required. Bayes' Theorem is **not** required.
	16.5 use permutation and combination to solve problems related to probability		
17. Measures of dispersion	17.1 understand the concept of dispersion	13	
	17.2 understand the concepts of range and inter-quartile range		

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	17.3 construct and interpret the box-and-whisker diagram and use it to compare the distributions of different sets of data		A box-and-whisker diagram can also be called a "boxplot".
	17.4 understand the concept of standard deviation for both grouped and ungrouped data sets		Students are required to recognise the term "variance" and that variance equals to the square of standard deviation. Students are required to understand the following formula for standard deviation: $$\sigma = \sqrt{\frac{(x_1 - \mu)^2 + \ldots + (x_N - \mu)^2}{N}} \ .$$
	17.5 compare the dispersions of different sets of data using appropriate measures		
	17.6 understand the applications of standard deviation to real-life problems involving standard scores and the normal distribution		
	17.7 understand the effect of the following operations on the dispersion of the data: (i) adding a common constant to each item of the set of data (ii) multiplying each item of the set of data by a common constant		

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18. Uses and abuses of statistics	18.1 recognise different techniques in survey sampling and the basic principles of questionnaire design	4	Students are required to recognise the concepts of "populations" and "samples". Students are required to recognise probability sampling and non-probability sampling. Students are required to recognise that, in constructing questionnaires, factors such as the types, wording and ordering of questions and response options influence their validity and reliability.
	18.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations		
	18.3 assess statistical investigations presented in different sources such as news media, research reports, etc.		
Further Learning Unit			
19. Further applications	Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learned in different areas	14	Examples: • solve simple financial problems in areas such as taxation and instalment payment • analyse and interpret data collected in surveys

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	The main focuses are: (a) to explore and solve more sophisticated real-life problems (b) to appreciate the connections between different areas of mathematics		• explore and interpret graphs related to real-life situations • explore Ptolemy's Theorem and its applications • model the relation between two sets of data which show a strong linear correlation and explore how to reduce simple non-linear relations such as $y = m\sqrt{x} + c$ and $y = k\,a^x$ to linear relations • explore the relation between the Fibonacci sequence and the Golden Ratio • appreciate the applications of cryptography • explore the Ceva's Theorem and its applications • analyse mathematical games (e.g. explore the general solution of the water puzzle)
20. Inquiry and investigation	Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts	10	This is **not** an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units.

Grand total: 250 hours